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Detection algorithm for turbulent interfaces and large-scale structures in intermittent flows



Jin Lee, Tamer A. Zaki*

Department of Mechanical Engineering, Johns Hopkins University, Baltimore, MD 21218, USA

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ABSTRACT

A robust algorithm is introduced for detection of large-scale coherent structures in transitional and intermittent flows that feature turbulent/non-turbulent (T/NT) interfaces. The algorithm is applicable the instantaneous flow fields of wall-bounded and free shear flows, and can effectively identify coherent events in the velocity or vorticity fields, or sweep/ejection motions. A database from direct numerical simulation (DNS) of transitional boundary layer is used to develop and demonstrate the capabilities of the algorithm which consists of three steps. The first is identification of the T/NT interface by comparing the normalized vorticity magnitude to a threshold value that is independent of the Reynolds number. The vorticity normalization is specifically designed to be applicable in transitional flows, where regions of the flow can host juxtaposed regions of laminar and turbulent flow. With the definition of the T/NT interface, conditional statistics are computed and perturbation quantities are defined relative to their respective conditional means. Second, the influence of the small-scale turbulence is excluded by applying an anisotropic Gaussian filter. The filter size is determined from the spatial characteristics of the smallscale vortical motions. In the third step, one-dimensional cores and two-dimensional surfaces within the flow structures of interest are identified from local extrema in the fields, and are tracked as Lagrangian objects. Using the algorithm, the population trends and advection speeds of large-scale sweep/ejection events are computed in the transitional boundary layer. Two additional flow configurations are also considered: turbulent jet flow emerging from a circular nozzle and the turbulent flow in a channel with a wavy surface.

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1. Introduction

At moderate and high-Reynolds numbers, a turbulent/nonturbulent interface separates chaotic fluid motion from laminar (or non-turbulent) fluid. The presence of the T/NT interface introduces a significant difficulty in the analysis of the turbulence and flow structures because the length and velocity scales change considerably across the interface [1]. In the turbulent region various contributions to the perturbation energy budget are of interest [2,3]. The classical interpretation of the turbulent spectrum is that dissipation takes place at the smallest vortical scales, and that energy is contained in the largest scales. The notion of scale motivates the study of turbulence structures. Analysis of these structures within the turbulent region has extended our understanding of the internal mechanism of turbulence [4]. In the non-turbulent region, interest is in the inception of laminar-turbulent transition in boundary layers [5], the far wake [1] and jets [6]. Detection of turbu-

* Corresponding author. E-mail address: t.zaki@jhu.edu (T.A. Zaki).

https://doi.org/10.1016/j.compfluid.2018.08.015 0045-7930/© 2018 Elsevier Ltd. All rights reserved. lence structures in intermittent flow regions is challenging because of the large gradients near the turbulent/non-turbulent interface. A robust algorithm is sought for the detection of large-scale flow structures near T/NT interfaces in intermittent flows, which is applicable to transitional and turbulent flows.

Several flow quantities have been adopted as a detector of the T/NT interface, for example the vorticity magnitude [1,7–10], the velocity–vorticity criterion [11,12] and the defect kinetic energy [13,14]. These identifiers were devised to detect the T/NT boundary at high-*Re*, and are not directly applicable to transitional flows. Depending on the detector function, the characteristics of the identified interface may vary slightly. However, the variations are rather immaterial when the turbulent region is adjacent to an irrotational, non-turbulent free stream. The same approach is, however, not directly applicable to transitional flows. For example, in transitional boundary layers, turbulence spots are fully surrounded by non-turbulent, but nonetheless vortical, region with strong mean-shear.

The large-scale flow coherence is usually defined by the long-wavelength modes of the streamwise velocity fluctuation u' [e.g. 15, 16]. In physical space, these structures are long regions

of u'. In transitional boundary layers, the structures are termed Klebanoff streaks [17,18] and large-scale and super structures in the turbulent boundary layer [19]. Detection of coherent turbulence motions in physical space has practical significance because a spectral, or Fourier, representation is not possible in complex geometries. Agrawal et al. [20] visualized the low-speed streaks from laser-induced fluorescence (LIF) images using noise filtering and thresholding. Dennis and Nickels [21] extracted the geometric center of large-scale u' motions, and reported the population trends of the negative and positive events separately. The local maximum of u' were also utilized to extract representative positions of the structure and to track them in space and time, in pretransitional [22,23] and in fully turbulent flows [24,25]. Each flow regime has been considered separately, and previous studies have not attempted to perform structure identification and tracking in the vicinity of the intermittent T/NT interface.

The large-scale coherence in shear flows is not limited to u' perturbations, and can also be observed in the Reynolds shear stress, u'v', and the streamwise vorticity, ω_x . The u' events have attracted much attention because they dominate the perturbation field, both visually and are the dominant contributor to the turbulence kinetic energy. The origin of these energetic motions is in the production term, where u'v' acts against the mean shear. In transitional boundary layer, the interaction has been described as tilting of mean vorticity [26] or lift-up [27]; In fully turbulent flows, the interpretation is in terms of ejection and sweeps [28]. The spatial correlation between u'v' and negative u' structures was examined by Lozano-Durán et al. [29] and Dennis and Nickels [21]. However, the spatial extent of the u'v' cross-correlation is shorter than that of the u' auto-correlation [30]. Similarly, Lee and Moser [16] noted that the emergence of the long-wavelength outer peak in the spectra of the shear stress lags in Reynolds number that in the streamwise stress event. Thus, it is important to analyze their role which requires a detection algorithm for the shear stress. Large-scale coherence in shear flows is also observed in the streamwise vortex, ω_x . Toh and Itano [31] reported that the large-scale streamwise swirl contributes to the self-sustaining turbulence cycle and the modulation of the near-wall structure. Also, the large-scale ω_x leads to the formation of the very-long streamwise perturbations [25], which motivates the identification of ω_x structures.

Image processing techniques have often been used to analyze flow field data, including noise reduction, edge detection and thresholding techniques. First, the noise, which corresponds to the small-scale turbulence, can be reduced by discarding highfrequency modes using grid coarsening or spatial filtering. [19] and [13] employed the box filter to exclude the small-scale feature for the large-scale u'-coherence and the T/NT interface. Previously adopted filtering techniques relied on a priori knowledge of the lengthscale of the flow and the features to remove. Edge detection, in image processing, relies on identifying the pixel with large gradient in brightness [32]. Similarly, the grid point where the flow quantities have specific gradients can be identified [24]. Lastly, thresholding is required for image segmentation. Otsu [33] and Prasad and Sreenivasan [34] provided methods for the gray-level picture thresholding, which inspired the algorithm for streak detection in pre-transitional boundary layers by Nolan and Zaki [22] and the T/NT interface detection by Holzner et al. [35]. In image processing, image brightness relies on the histogram of pixel value, which must be replaced by an appropriate measure in the context of fluid mechanics.

In the present study, we introduce a detection algorithm for large-scale coherence in intermittent flows with a T/NT interface. Structures defined by u', u'v' and ω_x in three-dimensional instantaneous flow fields (physical space) are detected. The algorithm is demonstrated in a transitional boundary layer, turbulent round jet and the flow over an undulating surface. Compared to previous

studies, the emphasis is placed on: (*i*) demarcation of the localized turbulent region (turbulence spot) from the mean shear, (*ii*) the anisotropic filtering by considering the small-scale turbulence, and (*iii*) application to various flow types and quantities. Ultimately, we present a detection algorithm for the large scales of turbulent quantities, which can be applied to both turbulent and intermittent flow fields seamlessly.

2. Sample dataset

A database from direct numerical simulation (DNS) of transitional boundary layer [36] is utilized to introduce the algorithm. This flow is challenging because it includes three T/NT interfaces: (i) an expanding interface separates spreading turbulent spots from their surrounding laminar flow; (ii) an interface separates the pre-transitional and fully turbulent boundary layer downstream; (iii) and an interface separates the fully turbulent boundary layer from the free stream. The computational domain starts upstream of the leading edge, which is a super ellipse defined by $\left(1 - \frac{1}{20L}x\right)^4 + \left(\frac{y}{L}\right)^2 = 1$ where *L* is the half thickness of the plate and the major-to-minor axes ratio is 20. In the body-fitted grid, the wall-parallel and wall-normal coordinates are ξ and η , respectively. The streamwise, vertical and spanwise directions are denoted x, yand z (Fig. 1), and corresponding velocity components are u, v and w. Throughout this work, the symbol $\langle \cdot \rangle$ denotes the ensemble average, while — denotes the unconditional average in time and in the homogeneous spanwise direction.

The incompressible Navier–Stokes equations were solved using a fractional step algorithm on a staggered grid with a local volume-flux formulation [37]. The viscous terms were integrated in time implicitly using the Crank–Nicolson method and the convective terms were treated explicitly using the Adams–Bashforth scheme. The spanwise Fourier transform of the pressure update is governed by a Helmholtz equation, which is solved for every spanwise wavenumber using multigrid in the $\xi - \eta$ plane.

Fig. 1 shows an instantaneous visualization of the flow field and the skin-friction coefficient. The size of the C-shape domain is 1100*L* (ξ), 40*L* (η) and 30*L* (*z* directions), and the length of the semi-infinite plate is 1050*L* in the *x* direction. The number of grid points is 4097 (ξ), 257 (η) and 257 (*z* directions). The boundary condition at the curved inlet (marked 'A' in Fig. 1(*a*)) was a superposition of a free-stream velocity U_{∞} and a fluctuating field from a separate computation of homogeneous isotropic turbulence u'_{HIT} . The intensity and integral length-scale of the isotropic turbulence at the curved inlet (x/L = -40) are $Tu = 0.03U_{\infty}$ and $L_k/L = 1.9$. The free-stream turbulence decays to $Tu = 0.02U_{\infty}$ at the leading edge (x = 0). The flat plate was a no-slip surface.

At the top and bottom boundaries (marked 'B' and 'C'), timedependent vertical velocities ensure zero-pressure-gradient. The suction velocity is computed using the continuity equation for a short-time averaged field and an active control. The controller uses a signal Δu_s which is the difference between the mean streamwise velocities of a sensor u_s and a desired value u_{target} ,

$$\Delta u_s(x,t) \equiv u_s(x,y_s,t) - u_{target}(x), \tag{1}$$

where y_s denotes the sensor position in the wall-normal direction. For the zero pressure gradient (ZPG), the target velocity is constant, $u_{target} = U_{\infty}$. The actual streamwise velocity u_s is obtained from the average over a duration *T*,

$$u_{s}(x, y, t) = \frac{1}{T} \frac{1}{L_{z}} \int_{t-T}^{t} \int_{0}^{L_{z}} u(x, y, z, t) dz dt.$$
 (2)



Fig. 1. (*a*) Instantaneous coherent structures in the transitional boundary layers and computational grid. (Color) Vortical structures identified by λ_2 ; (white) High- and (black) low-speed structures; (blue dashed) Boundary-layer edge. Every sixteenth line of the grid is shown for clarity. (*b*) Skin-friction coefficient as a function of streamwise position. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The suction boundary condition is then prescribed by,

$$\nu_{top}(x,t) = \underbrace{-\frac{\mathrm{d}}{\mathrm{d}x} \int_{0}^{L_{y}} u_{s}(x,y,t) \mathrm{d}y}_{\text{continuity}} + \underbrace{\varepsilon \Delta u_{s}(x,y_{s},t)}_{\text{active control}},$$
(3)

where ε denotes the control factor. We adopted the parameters $T = 10L/U_{\infty}$, $y_s = L_y - L$, and $\epsilon = 10^{-3} \sim O(1/Re)$.

At the outflow planes (marked 'D' and 'E'), convective boundary conditions are adopted. At the lower exit plane (marked 'E'), the mass-flow rate is adjusted to match that at the opposing cross-flow plane, 'F', on the upper surface. This ensures that the mean stagnation streamline ahead of the leading edge is horizontal. Periodic boundary condition is enforced in the spanwise direction. The Reynolds number $U_{\infty}L/\nu$ is 800, where U_{∞} and ν are the free-stream velocity and the kinematic viscosity. A video of the flow configuration and the flow field is available online [36].

Laminar-turbulent bypass transition takes place in the region 600 < x/L < 800, where C_f rises from the laminar level to turbulent correlation. For the present study, 1125 instantaneous flow fields were stored separated by $\Delta t = 1L/U_{\infty}$, and thus spanning $1124L/U_{\infty}$ time units.

3. Algorithm

The current algorithm consists of three steps: (i) conditional sampling, (ii) small-scale filtering and (iii) identification and tracking of the large-scale structures. The following is an overview of the algorithm.

- Step 1: conditional sampling
 - 1. Demarcation of T/NT regions
 - 2. Conditional statistics within the turbulent and nonturbulent regions
- Step 2: small-scale filtering
 - 1. Determination of the filter size
- 2. Application of the Gaussian filter to the turbulent fields
- Step 3: structure identification and tracking
 - 1. Identification of the position of the local extrema from the filtered fields

In contrast previous studies on fully-turbulent internal flow [e.g. 24, 25,38, 39], the present work on the intermittent flow pays particular attention to the first two steps. A precondition for utilizing the algorithm is access to converged conditional statistics, either computed during the simulation of from a sufficiently large number of stored instantaneous flow fields. Spatial homogeneity of the flow in one or more directions is helpful, although not a requirement of the algorithm.

3.1. Step 1: conditional sampling

The purpose of this step is to evaluate the fluctuating velocities, relative to an appropriate reference, or mean, within the turbulent and non-turbulent regions. First the T/NT interface is identified, in the context of transitional boundary-layer flow. Then, the conditional average is conducted to obtain the reference velocity and the perturbation field.

3.1.1. Detector function for the T/NT interface for high-Re – a brief review

Previous studies have adopted the magnitude of vorticity, $|\omega| \equiv (\omega_i \omega_i)^{1/2}$, to identify the irrotational boundary that encloses a turbulent region and separates it from an outer, non-vortical stream [e.g. 40]. Because the mean, and more generally the distribution, of $|\omega|$ varies with the streamwise position in a spatially-developing flow, a normalized vorticity was suggested by da Silva et al. [41]. Using such approach, a streamwise independent threshold of the normalized vorticity can be selected to identify that boundary. In free shear flows such as a jet, the normalization was performed using the root mean square (r.m.s.) $|\omega|^{rms}$, such that,

$$|\omega|^*(\mathbf{x}) \equiv \frac{|\omega|(\mathbf{x})}{\max\left(|\omega|^{rms}\right)(x)}.$$
(4)

The r.m.s. of the vorticity magnitude $|\omega|^{rms}$ is defined by the maximum in the cross-flow plane. In the wall-bounded flow, particularly zero-pressure-gradient turbulent boundary layer (TBL), a different normalization was proposed by Borrell and Jiménez [10],

$$|\omega|^* \equiv \frac{|\omega|}{u_\tau^2/\nu} \sqrt{\delta^+} = \frac{|\omega|}{u_\tau^2/\nu} \sqrt{\frac{u_\tau \delta}{\nu}},\tag{5}$$



Fig. 2. Probability density function (p.d.f.) of $\log_{10}|\omega|^*$ at (a) x/L = 900 (early turbulent) and (b) x/L = 200 (laminar region). (Dashed) $|\omega|^*_{th} = 0.2$; (dotted) normalized vorticity of the free-stream perturbation; (black) boundary-layer thickness δ .

where u_{τ} and δ are the friction velocity and the 99% boundarylayer thickness. Note that equation (5) was verified for turbulent boundary layer (TBL) at $\delta^+ \sim \mathcal{O}(10^3)$. In an earlier study, Taveira et al. [8] suggested the volume method to define the threshold for the irrotational boundary. That method identifies the value of the detector function where the volume of the turbulent region is least sensitive to changes in the threshold, and the wall-normal distance corresponding that threshold was larger than the conventional 99% thickness δ . A comparison of both approaches was provided by Lee et al. [42], and while both have their merits, there were both only attempted for turbulence beneath non-vortical streams. In the present study, however, we aim to identify the boundary that separates boundary-layer turbulence from a rotational outer flow, be that the streaky laminar region or the vortical free stream.

3.1.2. Identification of the interface in transitional flow

The detector function that was adopted to identify the irrotational boundary at high-Re (Section 3.1.1) must be modified for the identification of the interface in transitional flows. An attempt to identify the vorticity threshold using the volume method was not successful, because the free-stream flow is vortical and the volume of the turbulent region grows continually as the threshold is reduced, without a clear plateau. This issue is demonstrated clearly in Fig. 2, which shows the probability density function (p.d.f.) of $\log_{10}|\omega|^*$ at each y_w position. In the current dataset, y_w denotes the vertical distance from the wall. As shown in Fig. 2(a), the contours of the p.d.f. at x/L = 900 have two distinct regions relative to the threshold $|\omega|_{th}^* = 0.2$ (dashed line). One region centers around small values of $|\tilde{\omega}|^*$ at large- y_w , and the other near large values of $|\omega|^*$ at small-y_w region – the two regions being clearly associated with the free stream and the boundary layer, respectively. Recall that the $|\omega|^*$ -threshold has been designed to be streamwise independent at high-*Re* [10,42]. In the laminar region, x/L = 200(Fig. 2(b)), the boundary-layer vorticity at small y_w is larger than the threshold, which suggests that the flow is turbulent when in reality it is laminar and perturbed by Klebanoff streaks. In addition, the p.d.f. of vorticity in the free stream due to the weak background turbulence $(Tu \sim 0.01 U_{\infty})$ straddles the threshold. Two contributions to this shift are the presence of a weak level of vortical perturbation in the free stream and, more importantly, that the vorticity is artificially amplified when normalized by the small friction-velocity u_{τ} in the laminar region (see C_f in Fig. 1(*b*)). This result motivates a revising the normalization of the T/NT indicator function for transitional flows.

The interface detected by $|\omega|_{th}^* = 0.2$ is shown in Fig. 3(*a*). As discussed, the original definition of $|\omega|_{th}^*$ does not show a clear delineation of the free stream and the laminar and turbulent boundary layers. In order to improve the interface detection, we replace the friction velocity u_{τ} in equation (5) by the turbulent correlation,

$$u_{\tau,T} = 1.13 R e_{\theta}^{0.843} \nu / \delta, \tag{6}$$

which was provided by Schlatter and Örlü [43]. The normalization for the vorticity magnitude is modified accordingly,

$$|\omega|_m^* \equiv \frac{|\omega|}{(u_{\tau,T})^2/\nu} \sqrt{\frac{u_{\tau,T}\,\delta}{\nu}}.$$
(7)

Fig. 3(*b*) shows the isosurface of $|\omega|_m^* = 0.2$, which successfully eliminates the undesirable structures above the laminar boundary layer. However, the interface does not isolate the turbulent region, but rather also includes the laminar boundary layer upstream (blue isosurface in the figure), because the mean spanwise vorticity is included in the classical definition of the normalized threshold. Redefining the threshold based on the perturbation vorticity is not desirable because it requires prior knowledge of the flow statistics. In addition, the magnitude of the perturbation vorticity in the laminar region, e.g. due to the Klebanoff streaks, is appreciable which would hinder accurate prediction of the turbulent/non-turbulent boundary.

A possible improvement of the algorithm is to take advantage of the total streamwise vorticity, $\omega_x \equiv \partial w/\partial y - \partial v/\partial z$. At the wall, $|\omega_x|_w$ is insignificant in the laminar region relative to the turbulent boundary layer. A threshold for $|\omega_x|_w$ can therefore be established based on the p.d.f. of its logarithm. As shown in Fig. 4(*a*), the non-turbulent and turbulent regions flows are clearly separated in the streamwise direction with $|\omega_x|_{w,th} = 0.03U_{\infty}/L$ (dashed line). Fig. 4(*b*) shows the contour of $|\omega_x|_{w,th} = 0.03U_{\infty}/L$ in an instantaneous flow field. Dilatation and erosion are applied to $|\omega_x|_w$ in order to eliminate any holes, which yields the mask $M(|\omega_x|_w)$ in Fig. 4(*c*). This mask is used to refine the normalized vorticity field. The final form of the vorticity magnitude, which is applicable throughout this transitional flow, is denoted $|\omega|_t^*$ and defined by,

$$|\omega|_t^* = \begin{cases} |\omega|_m^* & \text{if } M(|\omega_x|_w) \ge |\omega_x|_{w,th}, \\ 0 & \text{otherwise.} \end{cases}$$
(8)

Fig. 3(*c*) shows the T/NT interface in the resulting transitional boundary layer. In comparison to the original approach that uses $|\omega|^*$, which was originally introduced for turbulent boundary layers, the current method accurately separates the laminar boundary layer from all regions of turbulence, including the turbulence spot at $x/L \approx 500$ in the figure (see also movie 1) for the time evolution of the interface identification. It is worth noting that the current definition for the vorticity magnitude yields identical results to $|\omega|^*$ in the fully turbulent region; i.e., $|\omega|_t^* = |\omega|^*$ when $u_{\tau,T} = u_{\tau}$ and $|\omega_x|_w/|\omega_x|_w$ th \gg 1. However, only the present approach can be applied indiscriminately throughout a transitional flow and accurately identify the interface surrounding any patches of turbulent



Fig. 3. Isosurface of the turbulent/non-turbulent (T/NT) interface at $t = t_0 + 500L/U_{\infty}$. (a) $|\omega|^* = 0.2$, (b) $|\omega|^*_m = 0.2$, (c) $|\omega|^*_t = 0.2$ conditioned with $|\omega_x|_w \ge 0.03U_{\infty}/L$.



Fig. 4. (a) Probability density function of $\log_{10}|\omega_x|_w$ at the wall. (Dashed) $|\omega_x|_{w,th} = 0.03U_{\infty}/L$. (b - c) Plan views of $|\omega_x|_w$ and $M(|\omega_x|_w)$ at $t = t_0 + 500L/U_{\infty}$. (Red) Turbulent and (white) non-turbulent regions identified using $|\omega_x|_{w,th}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in the intermittent flow regime and the fully turbulent flow down-stream.

3.1.3. The perturbation velocity field

Using the demarcation into non-turbulent and turbulent flows, conditionally averaged fields can be computed for each of the two regions. The conditional mean velocity profiles are plotted in Fig. 5(*a*, *b*). Data are shown only where the number of samples at each event exceeds 5% of the total. Within the transitional region (x/L = 700), for example, the mean velocities show nontrivial difference between the non-turbulent $\langle \cdot \rangle_{NT}$ and turbulent $\langle \cdot \rangle_{T}$ conditions. In Fig. 5(*a*), the streamwise component of the non-turbulent, or laminar, mean is smaller than the turbulent counterpart near the wall ($y_w/L = 0.5$), and vice versa away from the wall ($y_w/L = 4$). The difference in the mean vertical velocity (Fig. 5(*b*)) is more interesting, with $\langle v \rangle_{NT}$ and $\langle v \rangle_{T}$ having different signs in the transition zone. The non-turbulent contribution is negative in

the transition zone, which provides the necessary mean transport of streamwise momentum towards the wall in order to achieve a fuller $\langle u \rangle_T$ and satisfy the mean continuity equation. In the fully turbulent boundary layer region, (x/L = 900), intermittency persists near the boundary-layer edge, and again a notable difference in the conditional mean vertical velocities is observed. These differences between the non-turbulent and turbulent statistics highlights the importance of computing perturbation quantities relative to the appropriate means, in order to accurately identify low- and highspeed, or positive and negative, flow structures.

The fluctuating velocities relative to the non-turbulent $\langle u_i \rangle_{NT}$ and turbulent $\langle u_i \rangle_T$ mean are, respectively,

$$u_i'' = u_i - \langle u_i \rangle_{NT},\tag{9}$$

$$u_i' = u_i - \langle u_i \rangle_T. \tag{10}$$



Fig. 5. Conditional mean (*a*) streamwise and (*b*) wall-normal velocities. Conditionally sampled (*c*) m.s. streamwise velocity fluctuation and (*d*) Reynolds shear stress. (Red solid) Turbulent and (blue dashed) non-turbulent events; (gray) conventionally averaged quantity. The profiles are shown increasing values of $x/L = \{300, 500, 700, 900\}$, indicated with increasing line thickness. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The profiles of the mean square (m.s.) of the streamwise fluctuations, $\langle u''u'' \rangle$ and $\langle u'u' \rangle$, and the Reynolds shear stresses, $\langle u''v'' \rangle$ and $\langle u'v' \rangle$, are plotted n Fig. 5(*c*) and (*d*). The stresses computed relative to the unconditional mean, $u_i - \overline{u_i}$, are also shown for comparison (gray line). In the transition region (x/L = 700), the conditional stresses deviate appreciably from the classical Reynolds decomposition (gray). These results indicate that the non-turbulent and turbulent systems should be considered separately, and thus the original flow field is separated into two dynamical systems with perturbation velocities relative to their respective means.

3.2. Step 2: small-scale filtering

Small-scale features in the perturbation field obfuscate the large-scale coherent motions of interest, and therefore must be removed prior to the structure identification and tracking procedure. The filtering step results in a smoothed perturbation field, which must be calibrated to take into account the impact on the amplitude of the original signal. Note that the filtered field will be used only to identify the geometric features of the coherent motions. Evaluations of any fluid dynamical quantities, e.g. turbulence statistics, must use the original, unfiltered fields within the region occupied by the structure. Hereafter, the description of the algorithm is focused on the u'-event, while examples of other quantities will be shown in Section 3.3.

3.2.1. Filter size

In order to exclude small-scale features from the flow field, a Gaussian filter is adopted in all directions. The Gaussian filter preserves localized features, in comparison to a box filter [32], and is applicable in inhomogeneous directions and thus circumvents the limitations of spectral filters. For example, in boundary layers and jets, the streamwise and normal coordinates are not periodic and hence spectral filtering is not possible. Also, the typical eddy size varies in the normal direction [6].

An important parameter in the Gaussian filter is its size, σ_{x_i} . It must be selected based on the size of the flow structures of interest which can be determined from the two-point correlation [e.g. 30, 44]. In transitional flows, the correlation is computed con-

ditional on the flow being turbulent,

$$R_{\psi_T\psi_T}(\mathbf{r}_{\mathbf{x}}; \mathbf{x_{ref}}) = \frac{\langle \psi_T(\mathbf{x_{ref}})\psi_T(\mathbf{x_{ref}} + \mathbf{r}_{\mathbf{x}})|_{\Gamma(\mathbf{x_{ref}})=1}\rangle}{\psi_T^{rms}(\mathbf{x_{ref}})\psi_T^{rms}(\mathbf{x_{ref}} + \mathbf{r}_{\mathbf{x}})},$$
(11)

where $\psi_T \equiv \Gamma \psi$ for a general quantity ψ , and Γ is the instantaneous intermittency factor,

$$\Gamma(\mathbf{x},t) = \begin{cases} 1 & \text{if } |\omega|_t^* \ge |\omega|_{th}^* \text{ (turbulent)} \\ 0 & \text{if } |\omega|_t^* < |\omega|_{th}^* \text{ (non-turbulent).} \end{cases}$$
(12)

The correlation coefficient (11) is normalized by the turbulentconditioned root-mean-square, $\psi_T^{rms} \equiv \sqrt{\langle \psi^2 |_{\Gamma=1} \rangle}$. Fig. 6(*a*) shows $R_{u'u'}$, which represent the size of the large-scale structures. The size of the isosurface $R_{u'u'} = 0.5$ (green) is on the order of δ in the *x* direction and 0.5 δ in the *y* and *z* directions. Although the filtering algorithm utilizes the correlation in the turbulent zone only, Fig. 6(*a*i) reports $R_{u'u'} = 0.5$ in the transition region for completeness. There, the correlation is very-large in streamwise extent (>2 δ), and is reflective of the elongated nature of Klebanoff streaks. In the turbulent region, the structures are of the order 1 δ as shown in Fig. 6(*a*ii). An appropriate choice of the filter size, to remove small-scale fluctuations in the turbulent regime, should therefore be shorter than 1 δ . The ambiguity in this choice motivates evaluating the size of the small-scale vortical motions which should be filtered directly.

Fig. 6(*b*) shows the correlation coefficients of the signed swirling strength λ_2^* , which is a modification of the λ_2 -criterion [45],

$$\lambda_2^* \equiv c \ \lambda_2^{neg},\tag{13}$$

where *c* and $\lambda_2^{neg} (\equiv |\min(\lambda_2, 0)|)$ give the sign and the strength of the swirling motion, respectively. The sign is determined from,

$$c = \begin{cases} \omega_{x}/|\omega_{x}| & \text{if } \lambda_{2,x}^{neg} = \max\left(\lambda_{2,x}^{neg}, \lambda_{2,y}^{neg}, \lambda_{2,z}^{neg}\right) \\ \omega_{y}/|\omega_{y}| & \text{if } \lambda_{2,y}^{neg} = \max\left(\lambda_{2,x}^{neg}, \lambda_{2,y}^{neg}, \lambda_{2,z}^{neg}\right) \\ \omega_{z}/|\omega_{z}| & \text{if } \lambda_{2,z}^{neg} = \max\left(\lambda_{2,x}^{neg}, \lambda_{2,y}^{neg}, \lambda_{2,z}^{neg}\right) \end{cases}$$
(14)

where $\lambda_{2,x}^{neg}$, $\lambda_{2,y}^{neg}$ and $\lambda_{2,z}^{neg}$ denote the magnitudes of the 2-D swirling motions in the streamwise, wall-normal and spanwise directions. Because the streamwise and spanwise vortices are pre-



Fig. 6. Conditional two-point correlations of (*a*) the streamwise velocity fluctuation u' and (*b*) the signed swirling strength λ_2^* . The streamwise reference position is located at (i) $x_{ref}/L = 700$ and (ii) $x_{ref}/L = 900$. The vertical reference position is $y_{w,ref} = 0.5\delta(x_{ref})$. Isosurfaces are shown at R = -0.1 (white), 0.3 (cyan) and 0.5 (green). Line contours are shown from R = 0.3 to 0.9 with an increment of 0.1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

dominant near the wall and the outer region, respectively, the dimension of $R_{\lambda_2^*\lambda_2^*}$ represents the typical sizes of the streamwise and spanwise vortices in those two regions. Fig. 6(*b*), evaluated at $y_{w,ref} = 0.5\delta$ (outer region), thus shows the size of the spanwise vortex. At $R_{\lambda_2^*\lambda_2^*} = 0.5$ (green), the streamwise extent is an order of 0.1 δ . Also, the difference in the streamwise dimension of $R_{\lambda_2^*\lambda_2^*} = 0.5$ in the transitional and turbulent regions is not as significant as the change in $R_{u'u'}$. In this work, the lengthscale of the signed swirling strength λ_2^* is used to set the size of the Gaussian filter. Specifically the lengthscale of $R_{\lambda_2^*\lambda_2^*} = 0.05$ is adopted. This iso-level is a typical choice to extract the size of the correlation, and has been used for example by Monty et al. [44] to identify the size of large-scale structures when $R_{u'u'} = 0.05$.

The two-point correlation $R_{\lambda_2^*\lambda_2^*}$ was evaluated throughout the transitional boundary layer. Fig. 7 shows the resulting filter size σ_{x_i} for this dataset. Unlike previous studies which assumed isotropy of the small-scales [e.g. 24], asymmetry of the Gaussian filter is taken into account in the present algorithm. Owing to the statistical homogeneity of the sample dataset in the span, the filter size is defined in 2-D anisotropic values, $\sigma_{x_i}(x, y)$ with i = 1, 2 and 3. In general, the determined filter size inside the boundary-layer edge (dashed line in Fig. 7) increases towards the free stream at downstream as *Re* increases. The filter sizes vary up to approximately $2L(\approx 0.2\delta)$ near the downstream end of the computational domain. The full width at half maximum (FWHM) of the Gaussian function is close to the spanwise cut-off wavelength used by Bernardini and Pirozzoli [46], which was $\lambda_z = 0.5\delta$. In contrast to the upstream non-turbulent region, the turbulent flow $(x/L \gtrsim 700)$ has a signifi-

cant reduction of the filter size near the wall due to the generation of the turbulent near-wall streaks.

3.2.2. Gaussian filter in the Cartesian coordinates

The flow fields, especially data from numerical simulations, are often represented on non-uniform grids (e.g. Fig. 8(*a*), (*b*)). In order to apply the Gaussian filter, non-uniformity in grid spacing and edge handling should be taken into account. Separability of the Gaussian is used to define the kernel K_{x_i} at a given position **x**,

$$K_{x_{i}}(x_{i}^{p}, \mathbf{x}) = \exp\left(-\frac{(x_{i}^{p} - x_{i})^{2}}{2\sigma_{x_{i}}(\mathbf{x})^{2}}\right) \Delta V(x_{i}^{p}, x_{j}, x_{k}) \alpha(x_{i}^{p}, x_{j}, x_{k}) C_{x_{i}}(\mathbf{x}),$$
(15)

where \mathbf{x}^p are the surrounding points within the kernel. For an asymmetric Gaussian [47],

$$\sigma_{x_i} = \mathcal{H}(x_i^p - x_i)\sigma_{+x_i} + \mathcal{H}(x_i - x_i^p)\sigma_{-x_i}$$
(16)

where \mathcal{H} is the Heaviside function. Note that σ_z is symmetric in the sample dataset, $\sigma_z = \sigma_{+z} = \sigma_{-z}$. In equation (15), the cell volume, $\Delta V (\equiv \Delta x \Delta y \Delta z)$, takes into account the non-uniform grid spacing. For edge handling, the coefficient α is set to either unity inside the flow domain or zero outside the domain. The coefficient C_{x_i} is determined to ensure that the Gaussian kernel, K_{x_i} , is appropriately normalized to unity, $\sum_{x_i^p} K_{x_i}(x_i^p, \mathbf{x}) = 1$.

The Gaussian smoothing is completed by taking the convolution of the instantaneous flow field and the kernel. For a general quantity ψ , the output of the linear spatial filter, or the filtered quantity $\hat{\psi}$, is given by

$$\widehat{\Psi} \equiv \left(\left(\psi * K_z \right) * K_y \right) * K_x, \tag{17}$$



Fig. 7. Size of the Gaussian filter measured by $R_{\lambda_{2}\lambda_{2}^{*}} = 0.05$. (*a*) Downstream size σ_{+x} , (*b*) upstream size σ_{-x} , (*c*) upward size σ_{+y} , (*d*) downward size σ_{-y} and (*e*) symmetric spanwise size σ_{+x} . The boundary-layer thickness δ is marked by the dashed line.



Fig. 8. Sketch of the two-dimensional Gaussian kernel. (a) Cross-stream plane in Cartesian coordinates; (b) side view with the wavy-shaped immersed body (gray); (c) cylindrical coordinates.

where * denotes the convolution operator, $A * B = \sum_{x_i^p} A(x_i)B(x_i^p - x_i)$. Because the linear spatial filter is commutative, the order of 1-D convolutions does not affect the outcome. Unlike to the multidimensional Gaussian filter which requires nested loops in order to perform the convolution at a given position, the present separated filter achieves filtering using three independent loops. Where the kernel size is large, for example near the edge of the boundary layer in the present configuration, the use of three onedimensional Gaussian filters can greatly increase the computation efficiency relative to a single three-dimensional kernel.

3.2.3. Gaussian filter in cylindrical coordinates

A Gaussian filter that adopts multiple one-dimensional kernels is not adequate in cylindrical coordinates. For a dataset in cylindrical coordinates, the Gaussian function cannot be expressed along radial r grid lines near the pole (Fig. 8(c)). Therefore, a two-dimensional Gaussian filter is adopted in the radial-azimuthal (r, θ) plane in cylindrical coordinates, despite being computationally

less efficient,

$$K_{r\theta}(r^{p},\theta^{p};\mathbf{x}) = \exp\left(-\frac{(r^{p}-r)^{2}}{2\sigma_{r}(\mathbf{x})^{2}} - \frac{(r^{p}\theta^{p}-r\theta)^{2}}{2\sigma_{\theta}(\mathbf{x})^{2}}\right)$$
$$\Delta V(\mathbf{x},r^{p},\theta^{p}) \alpha(\mathbf{x},r^{p},\theta^{p}) C_{r\theta}(\mathbf{x}),$$
(18)

where the position vector $\mathbf{x} \equiv (x, r, \theta)$. In the axial direction, the asymmetric Gaussian function is used. Definitions of all other quantities are identical to the explanation in Section 3.2.2. The convolution of the flow field and the Gaussian kernel is,

$$\psi = ((\psi * K_{r\theta}) * K_x). \tag{19}$$

Allocating memory for large 2-D kernel and computing the convolution reduces computational efficiency. However, the application of the 2-D Gaussian filter on the original grid is more accurate than interpolation onto a Cartesian grid and filtering in three consecutive directions.

3.2.4. Calibration of the filtered perturbation field

The spatial filter reduces the magnitude of the fluctuating signal when eliminating high-frequency modes. To compensate for



Fig. 9. Streamwise velocity fluctuations in the cross-flow plane at x/L = 700 and $t = t_0 + 500L/U_{\infty}$. (*a*) Fluctuations relative to the non-turbulent mean u'/U_{∞} , (*b*) fluctuations relative to the turbulent mean u'/U_{∞} , (*c*) filtered turbulent fluctuations after calibration $\tilde{u'}/U_{\infty}$. Black line denotes (solid) 0.1 and (dashed) -0.1. Green solid line marks the T/NT interface. (*d*) The calibration function $f_{\hat{u'}/u'}$ at x/L = 700. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

this attenuation, the filtered signal $\hat{\psi}$ should be calibrated. Note that the calibrated signal will only be used to identify the large-scale motions, and is not used to compute any flow statistics; these are evaluated based on the original unfiltered fields only. The calibrated filtered field is given by,

$$\psi \equiv \psi / f_{\widehat{\psi}/\psi},\tag{20}$$

where the correction factor $f_{\widehat{\psi}/\psi}(\mathbf{x}) \equiv \widehat{\psi}^{rms}/\psi^{rms}$ is inhomogeneous in space due to the inhomogeneity of the filter size. The calibrated filtered field $\widetilde{\psi}$ thus preserves the same root-mean-square level of the original fluctuation field ψ . Since filtering is only applied in the turbulent region, this calibration is not relevant to the non-turbulent flow where $\widetilde{\psi} = \widehat{\psi} = \psi$.

An example of the small-scale filtering of *u*-fluctuations in the transitional boundary layer is shown in Fig. 9. In the cross-stream plane, the non-turbulent perturbations u'' (Fig. 9(*a*)) reflect the shape of laminar Klebanoff streaks outside of the T/NT interface (green line). In contrast, the fluctuations of the turbulent system u' (Fig. 9(*b*)) include small-scale feature within the local intermittent regions of turbulence. These small-scales lead to noise in the detection of large-scale motions [32]. After applying the filtering and calibration, Fig. 9(*c*) shows the shape of turbulent u'-structures which retains the large-scale features only. The calibration function $f_{u'(u')}$ is provided in Fig. 9(*d*).

3.3. Step 3: extraction of cores

The final step is to identify the spatial coordinates of the flow structure of interest within the filtered field. The core of the structure is detected by identifying the local extrema of the field and establishing their connectivity [22]. Connectivity in the streamwise direction leads to a one-dimensional skeleton, and connectivity in the streamwise-vertical plane leads to a two-dimensional surface that bisects the object. The latter is important, for example, when evaluating the intersection of large-scale motions with wallparallel planes or their extent at various heights in wall-bounded flows [48]. Tracking in time can be performed by cross-correlating successive flow fields.

The spatial coordinates of the cores of positive and negative structures are denoted $\mathbf{x}_{+\tilde{\mathcal{W}}}^{s}$ and $\mathbf{x}_{-\tilde{\mathcal{W}}}^{s}$.

$$\mathbf{x}_{+\widetilde{\psi}}^{s} = \mathbf{x} \quad \text{if } \partial \widetilde{\psi} / \partial z = \partial \widetilde{\psi} / \partial y = 0 \text{ and } \widetilde{\psi} > \psi_{th}$$
(21)

$$\mathbf{x}_{-\widetilde{\psi}}^{s} = \mathbf{x} \quad \text{if } \partial \widetilde{\psi} / \partial z = \partial \widetilde{\psi} / \partial y = 0 \text{ and } \widetilde{\psi} < -\psi_{th}.$$
 (22)

The superscript *s* is a unique identifier of each structure, and the threshold ψ_{th} facilitates isolating high-amplitude events. Similarly, the two-dimensional surface bisecting the structure is defined by $\mathbf{x}_{+,i\tilde{\nu}}^{p}$ and $\mathbf{x}_{-i\tilde{\nu}}^{p}$, where

$$\mathbf{x}^{p}_{+\widetilde{\psi}} = \mathbf{x} \quad \text{if } \partial \widetilde{\psi} / \partial z = 0 \text{ and } \widetilde{\psi} > \psi_{th}$$
 (23)

$$\mathbf{x}^{p}_{-\widetilde{\psi}} = \mathbf{x} \quad \text{if } \partial \widetilde{\psi} / \partial z = 0 \text{ and } \widetilde{\psi} < -\psi_{th}.$$
(24)

An example of the detection of large-scale *u*-structures is shown in Fig. 10 - see also movie 2 for the temporal evolution. In this case, the quantity ψ in equations (21)–(24) is u' for the turbulent system and $u''(\equiv u'')$ for the non-turbulent region, and the threshold is $u_{th} = 10\%$ of the free-stream speed U_{∞} . The original field (Fig. 10(a)) clearly shows the pre-transitional Klebanoff streaks, localized turbulent patches near the spanwise edges of the domain, and a fully turbulent flow downstream. The filtered field in the second panel excludes the small fluctuations while preserving the large-scale features. The detected cores of the streaks are shown in Fig. 10(c), with low- and high-speed structures distinguished by the colormap. The streamwise connectivity shows the elongated spines of the Klebanoff distortions in the laminar boundary layer, and of the large-scale motions in the turbulent flow. The former are very long in the streamwise direction and reside away from the wall (dark color), while the streaky structures in the turbulent region are relatively short and reside nearer to the wall (light color). The current algorithm thus detects both the outer



Fig. 10. Detection of large-scale streaks in transitional boundary layer at $t = t_0 + 500L/U_{\infty}$. Isosurfaces of (*a*) streamwise velocity fluctuations $u'\Gamma + u''(1-\Gamma) = \pm u_{th}$; (*b*) filtered fluctuations $\tilde{u'}\Gamma + u''(1-\Gamma) = \pm u_{th}$; (*c*) detected cores $\mathbf{x}_{\pm u'}^s$; (*d*) detected surfaces $\mathbf{x}_{\pm u'}^p$. The adopted threshold is $u_{th} = 0.1U_{\infty}$. Green solid line marks the T/NT interface on the wall. Transparent isosurfaces are structures in the non-turbulent region, $\Gamma = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

large-scales and the near-wall streaks simultaneously. Fig. 10(*d*) shows the surface bisecting the streaks along their spanwise maxima, \mathbf{x}^p . Unlike the core \mathbf{x}^s , this representation also includes the connectivity in the wall-normal direction, and shows that large-scale *u*-structures identified by $u_{th} = 0.1U_{\infty}$ are generally attached to the wall.

Other flow structures of interest include streamwise vorticity, streamwise vortices, and structures associated with sweep and ejection events [25,29,31]. Fig. 11 shows a coherence of the total streamwise vorticity, which corresponds to $\psi = \omega_x$ in equations (21-22) – see also movie 3 for the temporal evolution. Unlike the original field in the top panel, where the coherent streamwise vorticity is difficult to discern, the detected core in Fig. 11(c)clearly distinguishes the location and shape of these structures. The lengths of ω_x events are generally shorter than of u' (compare Figs. 11(c) and 10(c)). In the laminar boundary layer, streamwise vorticity is negligible because the dominant perturbations are Klebanoff steaks, or wall-normal vorticity [26,49]. The secondary instability of Klebanoff streaks, however, involves the generation of streamwise vorticity [50]. The cores of ω_x in the transition region thus have a meandering, helical appearance. In the fully turbulent boundary layer, the streamwise-vorticity structures are adjacent in the span to the cores of u' and have a similar appearance.

Sweep and ejection motions have a significant role in the production of turbulent kinetic energy through their contribution to the Reynold shear stress. Their organization in sweep-ejection pairs and a vortex cluster was reported by Lozano-Durán et al. [29]. The current algorithm can be used to identify these flow structures, and can distinguish ejections (Q2) and sweeps (Q4) using the following expression,

$$Q' = \begin{cases} u'v' & \text{if } u' < 0 \text{ and } v' > 0 \text{ (Q2; ejection)} \\ -u'v' & \text{if } u' > 0 \text{ and } v' < 0 \text{ (Q4; sweep)} \\ 0 & \text{otherwise.} \end{cases}$$
(25)

A similar definition of Q'' in the non-turbulent region uses u''and v''. Both Q' and Q'' are negative for ejections and positive for sweeps. The cores of these structures are identified in Fig. 12 – see also movie 4 for the temporal evolution. The adopted threshold was $Q_{th} = 0.002U_{\infty}^2$. Inside the young turbulent spots in the transition zone, and also within the tail of the fully turbulent boundary layer, the ejection motions are more prevalent. Within the fully turbulent flow, streamwise coherence in the Reynolds shear stress events exceeds 2δ (or ~ 20L at the exit plane). The spectral signature of these structures is a peak in the premultiplied co-spectra of u' and v', which was reported for example by Balakumar and Adrian [3].

3.4. Analysis of the detected structures

The present algorithm can be exploited to characterize the detected structures, for instance their sizes, advection velocities and intensities. Here, the coherent Reynolds shear stress events in the transitional boundary layer will be examined. As noted in §3.2, the original unfiltered fields are used in the analysis, for example to compute flow statistics, within the detected structures.



Fig. 11. Detection of the large-scale streamwise vorticity in transitional boundary layer at $t = t_0 + 500L/U_{\infty}$. Isosurfaces of (*a*) streamwise vorticity $\omega_x = \pm \omega_{x,th}$; (*b*) filtered streamwise vorticity; (*c*) the position of detected cores $\mathbf{x}_{+\omega_x}^s$ (red) and $\mathbf{x}_{-\omega_x}^s$ (blue). The adopted threshold is $\omega_{x,th} = 0.1U_{\infty}/L$. Green solid line marks the T/NT interface on the wall. Transparent isosurfaces are structures in the non-turbulent region, $\Gamma = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



Fig. 12. Detection of large-scale (red) sweep and (blue) ejection events in transitional boundary layer at $t = t_0 + 500L/U_{\infty}$. Isosurfaces of (*a*) original fluctuation $Q'\Gamma + Q''(1 - \Gamma) = \pm Q_{th}$; (*b*) filtered fluctuation $\widetilde{Q'}\Gamma + Q''(1 - \Gamma) = \pm Q_{th}$; (*c*) the position of detected cores, $\mathbf{x}_{Q'}^s$. The adopted threshold is $Q_{th} = \pm 0.001U_{\infty}^2$. Green solid line marks the T/NT interface on the wall. Transparent isosurfaces are structures in the non-turbulent region, $\Gamma = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.4.1. Population trends of the large-scale sweeps and ejections

The procedure which led to Fig. 12(*c*) was repeated for the entire database of flow fields, and population trends of turbulent sweeps and ejections were evaluated. Two properties were considered: the strength $\langle u'v'\rangle^c$ and the length l_{x_i} of the structures (see schematic in the inset of Fig. 13(*a*)). The strength is evaluated as the average of the shear stress along the core of the structure,

 $\langle u'v' \rangle^c \equiv \frac{1}{N} \sum_{n=1}^{N} u'v'(\mathbf{x}_{Q2(4)}^s(n))$, where *N* is the number of points along the core. The lengths l_x and l_y denote the extent of the structure in the streamwise and the wall-normal directions. The samples were binned into *x* and *y* based on the most upstream position of each structure, marked by the green square in the inset.

Fig. 13 shows the joint p.d.f. of the wall-normal distance and the averaged magnitude of sweeps and ejections along the core



Fig. 13. Joint p.d.f. of Q^- core strength and wall-normal distance, $\mathcal{P}(\langle u'v' \rangle^c, y_w)$, at (a) x/L = 700 and (b) x/L = 900. (Flood) Q2 and (line) Q4 cores. Contour levels increase logarithmically.



Fig. 14. Joint p.d.f. of Q^- core size and wall-normal distance, $\mathcal{P}(l_{x_i}, y_w)$, at (i) x/L = 700 and (ii) x/L = 900. (a) Streamwise length l_x ; (b) wall-normal height l_y . (Flood) Q2 and (line) Q4 cores. Contour levels increase logarithmically.

of the structure, $\mathcal{P}(y_w, \langle u'v' \rangle^c)$. Note that only structures longer than 0.5δ in the streamwise direction were conditionally sampled. Regardless of the streamwise position, the joint p.d.f.s have large values near the wall, which means the large-scale sweep/ejection motions are more frequently wall-attached than detached. In general, large-scale ejections exhibit a higher amplitude than sweeps, which is consistent with the findings from quadrant analyses that include all the flow scales [28], The only exception is in the nearwall region, where the large-scale ejections have lower population density. This suggests that strong near-wall ejections are unlikely to be associated with large scales. Comparison of Fig. 13(a) and (b)highlights the difference between the transitional and turbulent regions of the flow. While transition onset is due to an outer instability which starts near the free stream [26,51], turbulence production is in the near-wall region of the fully turbulent boundary layer. As such, the joint p.d.f. is shifted in Fig. 13(b) towards the wall.

Fig. 14(*a*) shows the joint p.d.f. of the streamwise length and the wall-normal elevation of the detected core, $\mathcal{P}(l_x, y_w)$. Considerable amount of very-long events (> 3 δ ; dashed line) are observed in the transitional region, and correspond to the Klebanoff streaks (Fig. 14(*a*i)). In the turbulent region (Fig. 14(*a*ii)), the streamwise extent of the structures is generally shortened, which is due to the generation of relatively shorter energetic turbulent motions. The

joint p.d.f. of the wall-normal size and elevation of the structures, $\mathcal{P}(l_y, y_w)$, is shown in Fig. 14(*b*). The majority of the population has a shorter wall-normal extent than 0.3 δ . The green horizontal line in the figure marks $y_w^+ < 20$. The joint p.d.f. of the wall-attached events, below this line, peaks near $l_y \approx 0.05\delta$, and therefore the inclination angle of the structures is not large. A slight increase in both l_y with increasing y_w is seen in the turbulent region (Fig. 14(*b*ii)), although not appreciable.

These results demonstrate the use of the structure identification algorithm to characterize the population of a quantity of interest. Further analyses are possible by tracking flow structures in timeand performing conditional statistical sampling of the flow fields.

3.4.2. Temporal analysis of large-scale sweeps and ejections

The time evolution of a large-scale ejection event (Q2) is shown in Fig. 15(*a*). The initial instance of the structure, at the arbitrary time origin $t = t_0$ (red in the figure), is wall-detached and nearly aligned in the streamwise direction. During the downstream evolution, it elongates and exhibits a larger inclination angle. Ultimately, the structure is divided into two downstream, at $t = t_0 + 80U_{\infty}/L$ (blue in the figure). Although the isosurface of u'v' (not shown) has a complicated geometry, the detected core is much simpler and clearly shows the temporal change.



Fig. 15. (*a*) Example of the temporal evolution of the large-scale ejection (Q2) core with the time increment of $20L/U_{\infty}$. Joint p.d.f. of (*b*, *c*) lifetime of the sweep/ejection core and the wall-normal position, $\mathcal{P}(T, y_w)$; (*d*, *e*) advection velocity of the core and the wall-normal position, $\mathcal{P}(u_c, y_w)$. (*b*, *d*) 650 < x/L < 750 and (*c*, *e*) 850 < x/L < 950 based on the emergence of the core. (Flood) The large-scale ejection and (line) the large-scale sweep events. Contour levels increase logarithmically.

Fig. 15(*b*), (*d*) shows the joint p.d.f. of the lifetime of the largescale sweeps/ejections and the wall-normal position, $\mathcal{P}(T, y_w)$. Because of the streamwise inhomogeneity of the flow, we only considered structures within the ranges 650 < x/L < 750 (Fig. 15(*b*)) and 850 < x/L < 950 (Fig. 15(*d*)). The wall-normal position is represented by the geometric center of the core at the first instance when the structure is identified, even though the wall-normal position of the core varies with downstream evolution. Note that the initial detection of structures encompasses the formation of entirely new ones or splitting of existing ones. The lifetime measures the duration between inception and either extinction or splitting. The result shows that most of the large-scale sweeps/ejections last less than $10U_{\infty}/L$. Nonetheless structures which have long lifetimes (e.g. larger than $20U_{\infty}/L$) are also observed.

Fig. 15(*c*), (*e*) shows the joint p.d.f. of the streamwise advection velocity of the cores (u_c) and the wall-normal position, $\mathcal{P}(u_c, y_w)$. To minimize statistical errors, only events with lifetimes larger than $4U_{\infty}/L$ were considered. The advected distance was evaluated from the coordinates of the geometric center of the core of the structure. Sweeps have a faster advection velocity relative to ejections, which is consistent with previous efforts that evaluated the advection velocities of the respective low- and high-speed structures [24].

4. Numerical examples

Two additional flow configurations are examined, and the performance of the algorithm for their analysis is demonstrated. The first is an emergent round turbulent jet, where both the confined flow in the nozzle and the spatially growing free jet are simulated. The second configuration is flow in a channel with a wavy bottom surface.

4.1. Turbulent round jet

The algorithm was applied to data from DNS of turbulent round jet. The streamwise (or axial), radial and azimuthal positions are denoted as *x*, *r* and θ , and the corresponding velocity components are u_x , u_r and u_θ (Fig. 16(*a*)). Based on the radius of the nozzle *R*, the axial extent of the computational domain is $6\pi R$ including the nozzle length which is πR . The radial extent of the domain is 8R. The number of grid point is $1153(x) \times 513(r) \times 385(\theta)$. The Navier–Stokes equations are solved in cylindrical coordinates, using the algorithm described by Jang et al. [52]. Turbulent inflow data were generated from an auxiliary computation of periodic turbulent pipe flow, at $Re_D \equiv (2R)U_b/\nu = 15000$ where U_b is the bulk velocity. In the jet simulation, the no-slip condition was applied to the nozzle surface. A convective outflow condition was prescribed at the exit plane of the computational domain, and all remaining boundaries were treated as impermeable free-slip surfaces.



Fig. 16. (*a*) Instantaneous visualization of the turbulent/non-turbulent (T/NT) interface in round jet at $Re_D = 15000$. Conditionally sampled (*b*) mean streamwise velocity $\langle u_x \rangle$, (*c*) streamwise stress and (*d*) Reynolds shear stress. (Solid) Turbulent and (dashed) non-turbulent events. The profiles are shown increasing values of $x/R = \{0, 5, 10\}$, indicated with increasing line thickness.

The position of the T/NT interface is shown in Fig. 16(a), which is identified using $|\omega|_{th}^* = 0.02$ throughout the laminar, transitional and turbulent regions of the jet. The vorticity magnitude of free shear flows is normalized using equation (4), and the threshold level is determined by the volume method [8]. Using the demarcated non-turbulent and turbulent regions, the conditional mean streamwise velocities, $\langle u_x \rangle_{NT}$ (dashed) and $\langle u_x \rangle_T$ (solid), were evaluated and are plotted in Fig. 16(b). Note that data is shown when the number of samples is greater than 5% of the total. The radial spreading of the turbulent region with downstream distance is evident in the conditional mean. In agreement with previous studies [e.g., 12], mean streamwise momentum is thus transported in the radial direction, and the amplitude of the velocity profile decreases downstream. Fig. 16(c) and (d) show the streamwise and the Reynolds shear stresses evaluated from the fluctuating velocities, relative to their conditional means.

Turbulent large-scale u'_x -structures which satisfy $u'_{x,th} = 0.1U_b$ were detected and are plotted in Fig. 17 and movie 5. Relative to the original data (Fig. 17(*a*), large-scale features are evident in the filtered field (Fig. 17(*b*)). The figure also highlights the scale growth of these structures with downstream distance. The detected cores (Fig. 17(*c*)) mark the local maximum of the fluctuating motion. The radial position of the cores is largely unaffected by the jet spreading within the computational domain. This observation is in agreement with Fig. 16(*c*), which shows almost similar radial position of the maximum streamwise stress. The detected surface (Fig. 17(*d*)) shows the very-large radial dimension of the structure. The present algorithm can be adopted for future studies of the role of large turbulent scales in the entrainment process and their contribution to flow statistics near the T/NT interface.

4.2. Wavy channel

The structure identification algorithm is applied to the turbulent flow in a channel with a wavy bottom topology that induces flow separation. The flow configuration is similar to the work by Cherukat et al. [53]. The bottom-wall undulation is a sinusoidal function with period 2h and amplitude 0.1h, where h is the averaged half-height of the channel. An orthogonal grid was generated using an elliptic grid generation algorithm [54]. The domain length and width are 16h and $2\pi h$, respectively, which are larger than previous studies, in order to accurately capture the large-scale features of the flow. The number of grid point is $1025(\xi) \times 385(\eta) \times 385(z)$. The numerical solution of the Navier-Stokes equations is identical to the description in \S 2. The Reynolds number is 3460 based on the bulk velocity U_b and h. The bulk flow was driven by the constant mass-flow-rate constraint. No-slip conditions were applied on both the wavy (bottom) and flat (top) walls. Periodic boundary conditions were enforced in the streamwise and spanwise directions. Flow statistics were collected using averaging in the spanwise direction and time, and phase-averaging in the streamwise direction. For the purpose of structure identification, flow fields obtained from solution of the Navier-Stokes equations in the body-fitted grid system were interpolated onto a Cartesian grid (c.f. Fig. 8(b)).

Fig. 18 and movie 6 show the detected large-scale *u'*-structures. The original data, the filtered field, and the detected cores and surfaces are all plotted. The cores of the large scales are properly identified over both the flat wall and the undulating surface. Similarly to the boundary-layer flow, the streaks near the flat wall reach very long extent in the streamwise direction. Due to the lower-wall morphology, however, detected cores in that region are relatively short. Previous studies of flow over transverse ribs have shown that the outer flow becomes more energetic [55], and similarly in the present configuration very-long low-speed structure are observed above the wavy surface.

The black solid line in Fig. 18(c) marks the edge of the separation zone, which varies in the span beneath the high- and low-speed streaks. High-speed structures locally delay separation, while low-speed ones promote its onset. [56,57] observed the same be-



Fig. 17. Turbulent u'_x structures extracted from an instantaneous realization of turbulent round jet. (*a*) Streamwise velocity fluctuation with $u'_{x+th} = 0.1U_b$; (*b*) filtered fluctuation at $\widehat{u'_x} = \widehat{u'_{x+th}}$; (*c*) detected cores $\mathbf{x}^s_{\pm u'}$; and (*d*) detected surface $\mathbf{x}^p_{\pm u'}$ with $|\omega|^*_{th} = 0.02$. Only the left half of the domain is visualized for clarity. Red and blue are hightand low-speed structures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



Fig. 18. Detected u' structures extracted from an instantaneous realization of flow in a wavy channel. (*a*) Streamwise velocity fluctuation at $u'_{th} = 0.1U_b$; (*b*) filtered fluctuation at $\widehat{u'} = \widehat{u'}_{th}$; (*c*) detected cores $\mathbf{x}^{s}_{\pm u'}$. (*d*) detected surface $\mathbf{x}^{p}_{\pm u'}$. Red and blue are high- and low-speed structures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

havior in simulations of the flow in a compressor passage. The current algorithm can be used to quantify such observations, and to correlate the changes in separation with the overlying flow structures.

5. Summary

A detection and tracking algorithm was developed for largescale turbulent structures in transitional and intermittent flows with a turbulent/non-turbulent (T/NT) interface. Direct numerical simulation (DNS) datasets of transitional boundary layer, round jet and flow in a wavy channel were examined to introduce the algorithm and demonstrate its performance. The approach also naturally facilitates conditional sampling of flow statistics. An important feature of this algorithm is that it can be applied to flows with juxtaposition of laminar and turbulent regions, to identify coherent structures in physical space within each region. It is also applicable to inhomogeneous flows with any restriction, provided that sufficient samples are available to ensure convergence of flow statistics.

The algorithm consists of three elements. The first step is the evaluation of conditional statistics within the non-turbulent and turbulent regions. The T/NT discrimination is performed using a normalized vorticity magnitude, and our proposed normalization is applicable throughout transitional flows. The second step is eliminating the small-scale motions using anisotropic Gaussian filter. The filter size is determined using the typical dimension of the small-scale flow features, which is obtained from the conditional two-point correlation of the vortical strengths. The magnitude of the filtered field is normalized by the ratio of the r.m.s. levels between the filtered and original fields. The last step of the algorithm is to detect the cores of the structures using the local extrema in the field. Extraction of the simple geometry from the turbulent field facilitates the statistical analysis and tracking of the structures in time. As an example, the instantaneous flow fields of a transitional boundary layer were analyzed. The identification of large-scale coherent streamwise velocity perturbations, streamwise vorticity, and sweep/ejection motions were all demonstrated. Similar analyses were performed for a turbulent round jet and the flow in a wavy channel.

The present algorithm enables the study of the spatiotemporal evolution of flow structures. Combined with conditional sampling techniques, it can be used to establish the contribution of these structures to the dynamics. In addition to data from eddy resolving simulations, the algorithm is equally applicable to experimental data, for instance from tomographic particle image velocimetry (tomo-PIV) or magnetic resonance velocimetry (MRV).

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.compfluid.2018.08.015

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