Inverse Halftoning Using a Shearlet Representation

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ABSTRACT

In this paper, we present a new approach for inverse halftoning of error diffused halftones using a shearlet representation. We formulate inverse halftoning as a deconvolution problem using Kite *et al.*'s linear approximation model for error diffusion halftoning. Our method is based on a new M-channel implementation of the shearlet transform. By formulating the problem as a linear inverse problem and taking advantage of unique properties of an implementation of the shearlet transform, we project the halftoned image onto a shearlet representation. We then adaptively estimate a gray-scaled image from these shearlet-toned or shear-tone basis elements in a multi-scale and anisotropic fashion. Experiments show that, the performance of our method improves upon many of the state-of-the-art inverse halftoning routines, including a wavelet-based method and a method that shares some similarities to a shearlet-type decomposition known as the local polynomial approximation (LPA) technique.

Keywords: shearlets, wavelets, inverse halftoning, deconvolution

1. INTRODUCTION

Digital halftoning, sometimes referred to as *spatial dithering*, is a process of rendering a gray-scale image into a binary (black-and-white) image. Commonly used halftoning methods include white-noise dithering, blue-noise dithering, ordered dithering and error diffusion [1–4]. The process of recovering a gray-scaled image from a given halftone image is known as *inverse halftoning*. In this work, we will mainly focus on inverse halftoning techniques for halftone images based on error diffusion methods such as Floyed-Steinberg[3] and Jarvis *at el.* [2]. Some of the uses of inverse halftoning include applications to resizing, contrast enhancement, compression, removal of aliasing artifacts when displaying halftoned images on a non-printed medium, and digital archiving of old newspaper/articles. All of these processes can be improved if a gray-scaled version can be recovered before manipulation. It has been shown that error diffusion halftoning can be approximately modeled as a linear system[4] and thus the problem of inverse halftoning can be viewed as a special deconvolution problem. In this paper, we propose the use of a specially adapted shearlet-based deconvolution method to recover the gray-scaled image from its halftone. In the next section, we describe the linear approximation to the error diffusion halftoning process as described in [4].

2. LINEAR MODEL FOR ERROR DIFFUSION

Halftoning by an error diffusion model is based on the following non-linear process: Given the Floyd-Steinberg or the Jarvis error filter

$$h_{FS} = \frac{1}{16}\begin{bmatrix} 0 & \bullet & 7 \\ 3 & 5 & 1 \end{bmatrix}, \quad h_J = \frac{1}{48}\begin{bmatrix} 0 & 0 & \bullet & 7 & 5 \\ 3 & 5 & 7 & 5 & 3 \\ 1 & 3 & 5 & 3 & 1 \end{bmatrix},$$

the quantization error at $\bullet$ is diffused over a causal neighborhood according to the matrix values. More precisely, each pixel is identified in a raster-scan indexing scheme. Starting at the top-left pixel, the pixel's gray-scale value is made into a binary number by thresholding (1, if the value is greater than or equal to 1/2, and 0 otherwise). The quantization error is then diffused on neighboring pixels using the weights from h_{FS} or h_J and the next pixel

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in the raster-scan indexing is then made into a binary number. The process continues until the bottom-right pixel has been transformed. Note that the weights are non-zero only for those pixel locations that have not already been scanned, so that the diffusion never goes backward with respect to the scanning direction.

The error diffusion halftoning can be modeled as a convolution of the original gray-scaled image with a filter plus additive colored noise.[4] More precisely, for an image x of size $N \times N$, the halftone $y(n_1, n_2)$ can be expressed as follows:

$$y(n_1, n_2) = (p * x)(n_1, n_2) + (q * \upsilon)(n_1, n_2) \tag{1}$$

where $0 \leq n_1, n_2 \leq N - 1$, $*$ denotes convolution, p and q denote the impulse responses determined by the error diffusion model, and υ is the additive white noise. Equation (1) in the discrete Fourier transform (DFT) domain can be written as

$$Y(k_1, k_2) = P(k_1, k_2)X(k_1, k_2) + Q(k_1, k_2)\Upsilon(k_1, k_2),$$

where, for $-N/2 \leq k_1, k_2 \leq N/2 - 1$, $Y(k_1, k_2), P(k_1, k_2), X(k_1, k_2), Q(k_1, k_2)$, and $\Upsilon(k_1, k_2)$ are the 2D DFTs of y, p, x, q and υ, respectively. The transfer functions P and Q are defined by

$$P(k_1, k_2) = \frac{K}{1 + (K - 1)H(k_1, k_2)}$$

and

$$Q(k_1, k_2) = \frac{1 - H(k_1, k_2)}{1 + (K - 1)H(k_1, k_2)},$$

where $H(k_1, k_2)$ is the DFT of an error diffusion filter (i.e. the DFT of h_{FS} or h_J). The values $K = 2.03$ or $K = 4.45$ can be used for the Floyd-Steinberg or the Jarvis error filter, respectively.[4]

3. INVERSE HALFTONING VIA DECONVOLUTION

In inverse halftoning given a halftone $y(n_1, n_2)$ and knowing $p(n_1, n_2)$ and $q(n_1, n_2)$ the objective is to recover $x(n_1, n_2)$ from (1). Since, p and q are linear time invariant filters, recovering x from y can be viewed as a deconvolution problem. Once we invert the convolution operator P, the resulting aspect of the deconvolution problem can be viewed as a denoising problem in the presence of colored noise. This can be seen from the following:

$$\begin{aligned}\tilde{X}(k_1, k_2) &= \frac{Y(k_1, k_2)}{P(k_1, k_2)} \\ &= X(k_1, k_2) + \frac{Q(k_1, k_2)\Upsilon(k_1, k_2)}{P(k_1, k_2)},\end{aligned}$$

where the second term in the above equation represents the colored noise. Many methods have been proposed for denoising images in the presence of colored noise. In this paper, we employ shearlet-based denoising to obtain an estimate of the halftoned image.

4. THE SHEARLET TRANSFORM

The shearlet construction can be considered as a natural extension of wavelets into two-dimensions.[5] Its representative elements are defined by the two-dimensional affine system

$$\{\psi_{ast}(x) = |\det M_{as}|^{-\frac{1}{2}}\, \psi(M_{as}^{-1}x - t) :\, t \in \mathbb{R}^2\},$$

where

$$M_{as} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & \sqrt{a} \end{pmatrix}$$

is a product of a shearing and anisotropic dilation matrix for $(a, s) \in \mathbb{R}^+ \times \mathbb{R}$. The generating function ψ is such that

$$\hat{\psi}(\xi) = \hat{\psi}(\xi_1, \xi_2) = \hat{\psi}_1(\xi_1)\, \hat{\psi}_2\left(\frac{\xi_2}{\xi_1}\right),$$

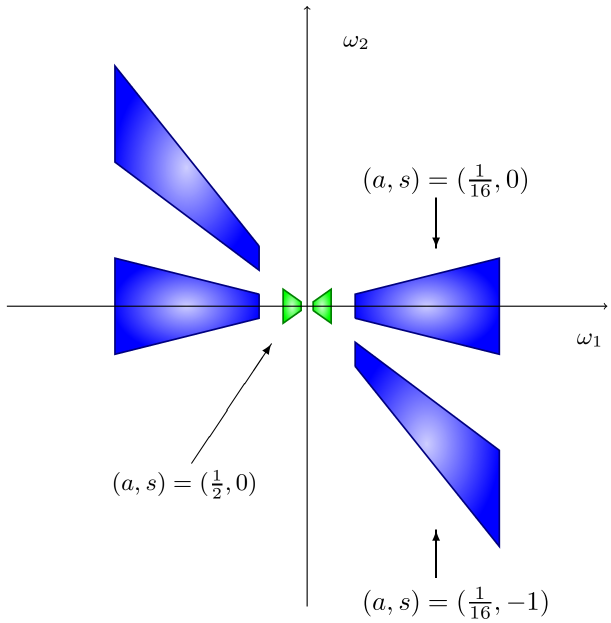

Figure 1. Frequency support of the shearlets for different values of a and s.

where ψ_1 is a continuous wavelet for which $\hat{\psi}_1 \in C^\infty(\mathbb{R})$ with supp $\hat{\psi}_1 \subset [-2, 1/2] \cup [1/2, 2]$, and ψ_2 is chosen so that $\hat{\psi}_2 \in C^\infty(\mathbb{R})$, supp $\hat{\psi}_2 \subset [-1, 1]$, with $\hat{\psi}_2 > 0$ on (-1,1), and $\|\psi_2\|_2 = 1$. Under these assumptions, a function $f \in L^2(\mathbb{R}^2)$ can be represented as

$$f(x) = \int_{\mathbb{R}^2} \int_{-\infty}^{\infty} \int_0^{\infty} \langle f, \psi_{ast} \rangle \, \psi_{ast}(x) \frac{da}{a^3} \, ds \, dt,$$

for $a \in \mathbb{R}^+$, $s \in \mathbb{R}$, and $t \in \mathbb{R}^2$. The operator S defined by $Sf(a, s, t) = \langle f, \psi_{ast} \rangle$ is referred to as the *continuous shearlet transform* of $f \in L^2(\mathbb{R})$. It is dependent on the scale variable a, the shear s, and the location t. An illustration of the spatial frequency support is shown in Figure 1.

The collection of *discrete shearlets* is given by

$$\{\psi_{j,\ell,k} = |\det A|^{j/2} \, \psi(B^\ell \, A^j x - k) : \, j, \ell \in \mathbb{Z}, k \in \mathbb{Z}^2\},$$

where

$$B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 0 \\ 0 & \sqrt{2} \end{pmatrix}.$$

Shearlets form a Parseval frame (tight frame with bounds equal to 1) for $L^2(\mathbb{R}^2)$ for the appropriate choices of generating function ψ.[6]

It is known that for a certain class of images that can be modeled as piecewise smooth functions that are smooth away from a C^2 edge (that is, a composite of a C^2 function plus an indicator function of a set whose boundary is C^2), wavelets do not yield the best possible non-linear approximation rate. Specifically, using the N largest wavelet coefficients, the approximation error for such an image decays as $O(N^{-1})$ as N increases. On the other hand, a representation such as shearlets or curvelets[7] yields the nearly optimal non-linear approximation rate of $O(N^{-2}(\log N)^3)$ as N increases. It is this faster decay rate that gives shearlets an improved performance over wavelets in estimating such images.

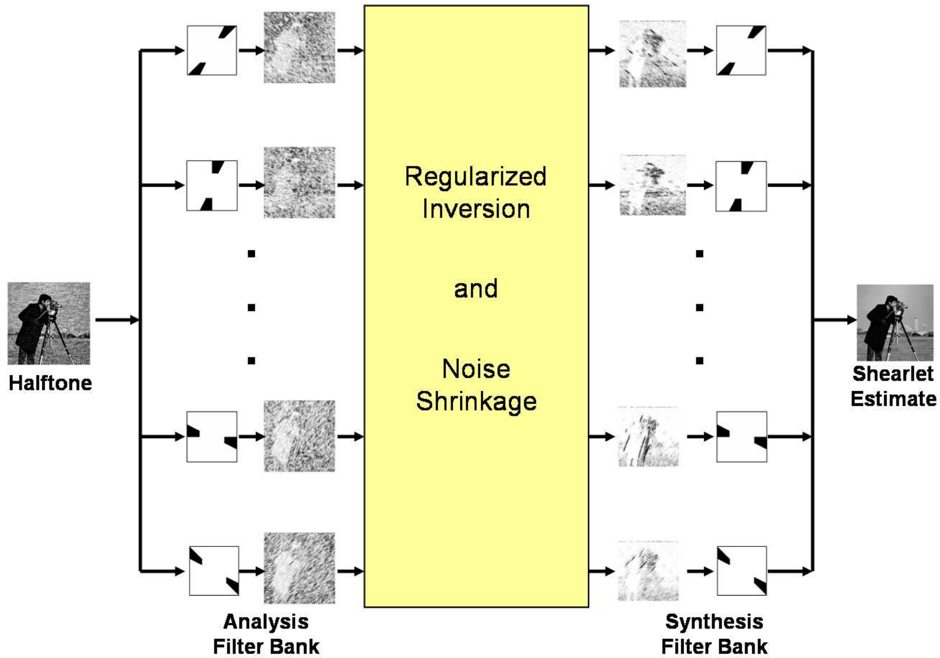

Figure 2. Shearlet-based inverse halftoning.

5. GENERALIZED CROSS VALIDATION FOR SHEARLET THRESHOLDING

In this section, we describe a shearlet-thresholding scheme based on a GCV function for the purpose of noise reduction[8]. One of the major advantages of this GCV method is that it obtains nearly the optimal thresholding without knowing the noise variance. It depends only on the data and automatically adjusts the shrinkage parameter according to the data. A similar GCV method for wavelet thresholding has been proposed in [9–11].

Suppose

$$\mathbf{y} = \mathbf{x} + \boldsymbol{\gamma},$$

where the vectors $\mathbf{y}$, $\mathbf{x}$ and $\boldsymbol{\gamma}$ represent respectively the observation, the original image and the colored noise that is assumed to be second order stationary (i.e. the mean is constant and the correlation between two points depends only on the distance between them). Corresponding to a threshold τ, define the soft-threshold function $T_\tau(x)$ to be equal to $x - \tau \text{sign}(x)$ if $|x| > \tau$ and zero otherwise.

If $\mathbf{y}_{j,\ell}$ represents the vector of shearlet coefficients of $\mathbf{y}$ at scale j and direction ℓ, then we can write

$$R(\tau) = \sum_j \sum_\ell \frac{L_{j,\ell}}{L} R_{j,\ell}(\tau_{j,\ell}),$$

where $L_{j,\ell}$ is the number of shearlet coefficients on scale j and direction ℓ, L is the total number of shearlet coefficients, and

$$R_{j,\ell}(\tau_{j,\ell}) = \frac{1}{L_{j,\ell}} \|T_\tau(\mathbf{y}_{j,\ell}) - \mathbf{x}_{j,\ell}\|^2.$$

The minimizer of $GCV_{j,\ell}(\tau_{j,\ell})$ is asymptotically optimal for the minimum risk threshold $R_{j,\ell}(\tau_{j,\ell})$ for scale j and directional component ℓ, where

$$GCV_{j,\ell}(\tau_{j,\ell}) = \frac{\frac{1}{L_{j,\ell}} \parallel T_\tau(\mathbf{y}_{j,\ell}) - \mathbf{y}_{j,\ell} \parallel^2}{\left[\frac{L_{j,\ell,0}}{L_{j,\ell}}\right]^2},$$

and $L_{j,\ell,0}$ is the total number of shearlet coefficients that were replaced by zero.[12] Thus, a good shearlet-based denoised estimate can be found by using the values $\tau_{j,\ell}$ that minimize $GCV_{j,\ell}$ for each j and ℓ.

6. INVERSE HALFTONING USING SHEAR-TONES

Having established a method for obtaining a good image estimate when the image is corrupted by colored noise, let us now focus on how we are to use this method as part of an inverse halftoning routine. Since our model is described by (1), a suitable estimate can be found by regularizing the convolution operator from a discrete Fourier basis. Using the regularized inverse operator

$$P_\alpha(k_1,k_2) = \frac{\overline{P}(k_1,k_2)}{|P(k_1,k_2)|^2 + \alpha^2|Q(k_1,k_2)|^2} \tag{2}$$

for some regularizing parameter $\alpha \in R^+$, an image estimate in the Fourier domain is given by

$$X_\alpha(k_1,k_2) = Y(k_1,k_2)P_\alpha(k_1,k_2),$$

for $-N/2 \le k_1, k_2 \le N/2-1$. This type of regularization is often referred to as *Tikhonov-regularization*. When an estimate of the power spectral density (PSD) can be accurately determined, a Wiener-based solution can be found by using

$$P_\alpha(k_1,k_2) = \frac{\overline{P}(k_1,k_2)P_{\hat{x}}(k_1,k_2)}{|P(k_1,k_2)|^2P_{\hat{x}}(k_1,k_2) + \alpha^2\sigma^2|Q(k_1,k_2)|^2} \tag{3}$$

where $\alpha \in \mathbb{R}^+$ and $P_{\hat{x}}(k_1,k_2)$ is the estimated PSD of the image for $-N/2 \le k_1, k_2 \le N/2-1$.

Taking advantage of the shearlet decomposition, we can adaptively control the regularization parameter to be the best suited of each frequency supported trapezoidal region. Let $G_{j,\ell}$ denote the DFT of the shearlet filter $g_{j,\ell}$ for a given scale j and direction ℓ. The shearlet coefficients of an estimate of the image for a given regularization parameter $\alpha_{j,\ell}$ can be computed in the Fourier domain as

$$X_{\alpha_{j,\ell}}(k_1,k_2) = Y(k_1,k_2)G_{j,\ell}(k_1,k_2)P_{\alpha_{j,\ell}}(k_1,k_2)$$

for $-N/2 \le k_1, k_2 \le N/2-1$.

The remaining aspect of the deconvolution problem is transformed into a denoising problem in the presence of colored noise. This can be dealt with by thresholding the estimated shearlet coefficients using the GCV determined previously without having to know the noise variance explicitly. Notice that $\{YG_{j,\ell}\}$ represents the halftoned image projected onto a shearlet representation (shear-tone elements) and that, in order to independently regularize and estimate from each shear-tone element, an M-channel filter bank decomposition of shearlets is needed.[12] An illustration of the shearlet-based method is shown in Figure 2.

There is a significant advantage in using the GCV for the shearlet thresholding, as the variance of the colored noise at each location and scale dependent on α does not have to be estimated. In addition, a GCV-based thresholding routine produces better results over schemes based on estimating the standard deviation of the noise throughout the decomposition.

We summarize the main steps of the shearlet-based inverse halftoning algorithm as follows:

Shearlet-based Inverse Halftoning Algorithm

Given $\alpha_{j,\ell}$, for some j and ℓ.

- Use the shearlet filter $G_{j,\ell}$ and apply the regularized filter (2) or (3) to Y to obtain $X_{\alpha_{j,\ell}}$.
- Apply the GCV based shearlet shrinkage to $x_{\alpha_{j,\ell}}$ to obtain $\widehat{x}_{\alpha_{j,\ell}}$.

Form the final estimate by applying the inverse shearlet transform.

In the demonstrations below, the regularization parameters $\alpha_{j,k}$ are considered as fixed design parameters for the purposed of inverse halftoning. In what follows, one unique set of design parameters has been used. Given the halftone image, it is possible to gain in performance if the parameters are chosen adaptively by using methods such as those suggested in [12].

7. EXPERIMENTAL RESULTS

In this section, we present results of our proposed algorithm and compare them with some of the recent multiscale wavelet and wavelet-like inverse halftoning methods described in [13][14]. In these experiments, we use the peak signal-to-noise-ratio (PSNR) to measure the performance of the routines tested. For an image of size $N \times N$ with $L + 1$ gray levels, the PSNR is defined as

$$PSNR = 20 \log_{10} \left(\frac{N \times L}{\|\hat{x} - x\|_2} \right),$$

where $\hat{x}$ denotes the estimated inverse halftoned image. For the shearlet transform implementation, we used 1, 8, 8, 16, and 16 directions in the scales from coarse to fine.[15] We apply the GCV based shrinkage to the outputs from each of the 48 filters except for the output corresponding to the coarsest scale. Experiments have shown that increasing the number of directions, every scale usually yields a better estimate.

We illustrate the performance of our inverse halftoning algorithm using a 512×512 *Lena* image and a 256×256 *Zebra* image halftoned using the Floyed-Steinberg algorithm.[3] The results of the first example on the Lena image are shown in Figure 3. The shearlet-based method yields a PSNR value 33.15 dB, which is better than the values obtained by the other methods. The close-up views of this experiment are shown in Figure 4.

In a second set of experiments, we applied our algorithm on the *Zebra* image. The results are shown in Figure 5. Again, the shearlet-based inverse halftoning algorithm outperforms the other methods in terms of PSNR.

8. CONCLUSION

We have proposed a new method of inverse halftoning using a shearlet representation. This is based on formulating the inverse halftoning problem as a deconvolution problem. Effective estimations are then made from this formulation by using a generalized cross validation function and an M-channel implementation of the shearlet transform. The results for this method demonstrate a state-of-the-art performance.

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(a) (b) (c) (d) (e)

Figure 3. Inverse halftoning experiment with the *Lena* image. (a) Original image. (b) Floyd-Steinberg halftone. (c) Wavelet-based estimate (PSNR = 31.96 dB). (d) LPA-ICI-based estimate (PSNR = 32.53 dB). (e) Shearlet-based estimate (PSNR = 33.15 dB).

(a) (b) (c) (d) (e)

Figure 4. Close-ups of inverse halftoning experiment with the *Lena* image. (a) Original image. (b) Floyd-Steinberg halftone. (c) Wavelet-based estimate (PSNR = 31.96 dB). (d) LPA-ICI-based estimate (PSNR = 32.53 dB). (e) Shearlet-based estimate (PSNR = 33.15 dB).

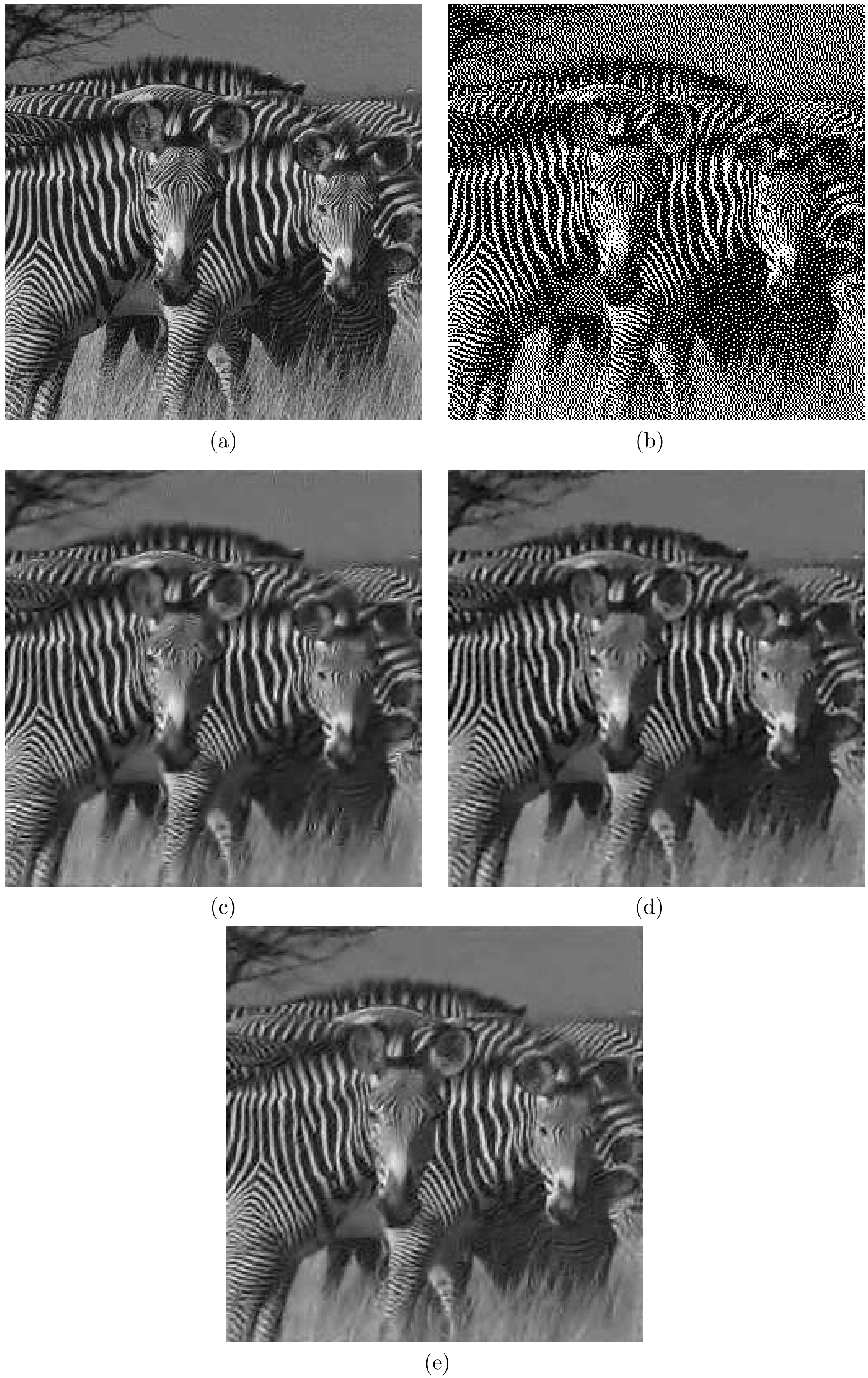

Figure 5. Inverse halftoning experiment with the *Zebra* image. (a) Original image. (b) Floyd-Steinberg halftone. (c) Wavelet-based estimate (PSNR = 23.72 dB). (d) LPA-ICI-based estimate (PSNR = 23.22 dB). (e) Shearlet-based estimate (PSNR = 24.22 dB).

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