A NEW MULTIRESOLUTION GENERALIZED DIRECTIONAL FILTER BANK DESIGN AND APPLICATION IN IMAGE ENHANCEMENT

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ABSTRACT

In this paper, we present an image enhancement technique based on a new multiscale generalized directional filter bank design. The design presented is a shift-invariant overcomplete representation, which is well suited to extracting geometric features such as edges. Special cases of this design method can be made to reduce to different and improved implementations of the shearlet and the contourlet transforms, which are known to represent certain classes of images optimally. Use of this new filter bank design has proven itself competitive in image restoration for noisy images and is well suited for distinguishing noise from weak edges. Experimental results show that our unique image enhancement technique out-performs wavelet and contourlet based enhancement methods.

Index Terms**—** Wavelet transforms, Image enhancement, Multidimensional digital filters, Image processing

1. INTRODUCTION

In image enhancement, the objective is to make the processed image better in some sense than the unprocessed image. One of the most well-known methods for contrast enhancement is histogram equalization which is useful for images with a poor intensity. Since edges contain important information about the image, they can be used to enhance the contrast. Many multiscale based image enhancement methods have been used in astronomical and medical imaging. In most of these methods, an image is decomposed in a separable fashion, not taking the advantage of the geometric information available in the subbands such as edges. It is well known that natural images have their energy strongly concentrated at low frequencies. Since directional information is related to mid and high frequency information such as edges and textures, lowpass information can hinder our ability to capture and analyze these features. Therefore, scale information needs to be used in conjunction with directional information in applications where it might be required to distinguish features of different sizes.

It has been observed that shift-invariance and redundancy are highly desirable in many image processing applications such as image denoising, edge detection, and image enhancement [1, 2]. In this paper, we exploit the directional and multiscale properties of this new shift-invariant generalized directional filter bank (GDFB) that is well suited for image enhancement. In Section 2, we discuss the key features of our new GDFB design method. In Section 3, we present the construction of the GDFB. We define a nonlinear mapping function for feature enhancement in Section 4 and present results in Section 5. Concluding remarks are made in Section 6.

2. GENERALIZED DIRECTIONAL FILTER BANK

The directional analysis employed in this paper decomposes the spectral region of a given image into the regions shown in Figure 1. This spatial-frequency tiling is equivalent to that of the discrete shearlet and contourlet transform. This means that this GDFB implementation could be considered as an alternate method for computing these transforms. In fact, the achieved filters can produce more precisely the desired frequency tiling than those exhibited in the NSCT (see Figure 3).

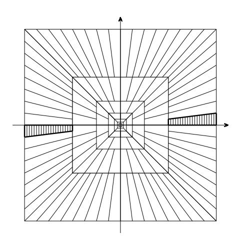

Figure 1: A representation of the spatial-frequency tiling used in this paper.

These filters satisfy a *parabolic scaling* so that each element in the Fourier domain is supported in a region whose width is approximately equal to the square of its length (see Figure 1). They also exhibit *highly directional sensitivity* in

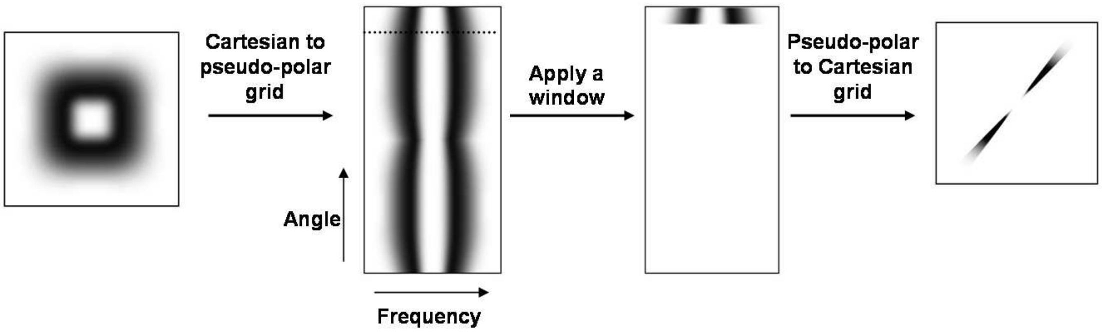

Figure 2: Analysis filter design flowgraph.

that each element is oriented along lines with slope $\ell\, 2^{-j}$ for $j \geq 0$,$-2^j \leq \ell \leq 2^j - 1$ and the number of orientations doubles at each finer scale. These properties are needed to essentially insure that a representation using these elements achieves the desired nonlinear approximation error rate for a certain class of images that can be represented as piecewise-smooth functions that are smooth away from discontinuities across smooth curves [3]. This means that retaining the N largest coefficients in this representation, the approximation error is bounded by $C\, N^{-2}\, (\log N)^3$, for some positive real constant C. By comparison, the approximation error for a wavelet representation is bounded by $C\, N^{-1}$ for the same class of images.

3. FILTER DESIGN

There are several filter design algorithms available in the literature for the design of multidimensional exact reconstruction filter banks. In general for m channels, the key is to design a set of analysis filters $\{H_i(\mathbf{z})\}_{i=0}^{m-1}$ and find a set of synthesis filters $\{G_i(\mathbf{z})\}_{i=0}^{m-1}$ satisfying the Bezout polynomial equation

$$\sum_{i=0}^{m-1} G_i(\mathbf{z}) H_i(\mathbf{z}) = 1. \qquad (1)$$

The most difficult challenge in designing a pair of analysis filters in higher dimensions for a filter bank is that in general the Bezout equation may not be solvable. When it is solvable, the standard algebraic techniques for solving the equation such as factorization are not applicable because no global factorization theorem exists. This means typical solutions rely on forming the higher-dimensional filters in a separable construction dependent on one-dimensional filter components.

To create the analysis filters, we first obtain a subband decomposition similar to that of the nonsubsampled Laplacian pyramid [4]. We then transform the LP data from Cartesian grid onto the pseudo-polar grid. As shown in Figure 2, a window is then applied in the pseudo-polar domain and the analysis filters are obtained in the frequency domain by inverting the pseudo-polar grid back to the Cartesian grid. Examples of directional filters created this way are shown in Figure 3. Filter designs that achieve complex geometrical spatial-frequency tiling are now possible using similar procedures.

In order to obtain the synthesis filters, we propose to use the solution techniques for the *Multichannel Deconvolution Problem* (MDP). In 1983, Berenstein *et al.* considered the following MDP: *Given a collection* $\{h_i\}_{i=0}^{m-1}$ *of finite impulse response filters on* $\mathbb{R}^d$ $(d \geq 2)$, *find a collection* $\{\tilde{h}_i\}_{i=0}^{m-1}$ *of finite impulse response filters such that* $\sum_{i=0}^{m-1} h_i * \tilde{h}_i = \delta$, *where* δ *is a Dirac delta function.* This equation in the Fourier-Laplace domain is known as the analytic Bezout equation and its discretization corresponds to (1). The recent methods for solving the MDP in a discrete setting provide a more effective way of constructing appropriate synthesis filters than the standard methods (see [5] for specific techniques and references). Thus, using these methods, we are no longer constrained in the traditional ways to create higher dimensional directional analysis and synthesis filters. For this paper we propose to use the solution methods provided in [5] to obtain the synthesis filters.

An advantage of a filter bank created this way is that it can project the image directly onto the desired coefficient basis. Furthermore, this type of filter bank implements a multiresolution and multidirectional decomposition of an image, and is a perfect reconstruction filter bank which can be implemented in $O(N^2 \log N)$ operations for an $N \times N$ image. The GDFB has a redundancy of $\sum_{k=0}^{J} 2^{l_k}$, where l_k denotes the number of directions at the kth scale of the LP.

4. IMAGE ENHANCEMENT

Several multiscale analysis based enhancement techniques have been developed [4, 6, 7, 8]. The goal of the nonlinear mapping function is to amplify weak edges and to sup-

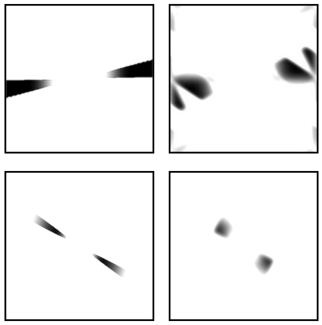

Figure 3: Images of Directional Filters. The images on the left correspond to examples of the frequency responses of the GDFB. The images on the right correspond to examples of the frequency responses of the NSCT.

press noise. Here, we use a new adaptive nonlinear mapping function that incorporates the nonnegative garrote shrinkage functions, which provide a good compromise between hard and soft shrinkage rules, in order to avoid amplifying noise and remove small noise perturbations (similar to [6]). We define this nonlinear operator as follows, using the notation $\mathrm{sigm}(y) = (1 + e^{-y})^{-1}$:

$$\begin{aligned} f(y) &= 0 \quad \text{if } |y| < T_1 \\ f(y) &= \mathrm{sign}(y)T_2 + \bar{a}(\mathrm{sigm}(c(g_y - b)) - \mathrm{sigm}(-c(g_y + b))) \\ &\quad \text{if } T_2 \leq |y| \leq T_3 \\ f(y) &= y \quad \text{otherwise} \end{aligned} \tag{2}$$

where $y \in [-1, 1]$, $\bar{a} = a(T_3 - T_2)$, $b \in (0, 1)$, c is a gain factor, $0 \leq T_1 \leq T_2 < T_3 \leq 1$, $g_y = \frac{\mathrm{garrote}_{T_2}(y)}{T_3 - T_2}$, where

$$\mathrm{garrote}_{T_2}(y) = \begin{cases} 0, & |y| \leq T_2 \\ y - \frac{T_2^2}{y}, & |y| > T_2 \end{cases}$$

and a can be computed by $a = (\mathrm{sigm}(c(1-b)) - \mathrm{sigm}(-c(1+b)))^{-1}$. Here b and c determine the threshold and rate of enhancement, respectively. As can be seen from Figure 4, the coefficients in $[T_2, T_3]$ are modified for enhancement while the coefficients in $[0, T_1]$ are suppressed. These parameters can be adaptively estimated by using the robust median operator [9] and the noise variance in each subband [10]. For example, T_1, T_2, and T_3 for the subband j can be chosen as $p\sigma_j\sigma, q\sigma_j, r\sigma_j$, respectively, where σ is the noise variance of the input image and σ_j is the noise variance of the jth subband and p, q and r are user defined values. Through this nonlinear function, the subband coefficients can be pointwise modified for image enhancement by $\tilde{y_k} = y_{k_{\max}} f\left(\frac{y_k}{y_{k_{\max}}}\right)$, where $1 \leq k \leq m$, y_k is the output of the kth channel of the filter bank, and $y_{k_{\max}}$ is the maximum absolute amplitude of y_k.

Our method for image enhancement using the GDFB consists of the following steps:

1. Estimate the noise standard deviation in the $N \times N$ input image using the robust median operator [9].

2. Pass the input image through the analysis part of the GDFB. At this point, we get a set of m subbands, each corresponding to a given resolution level. Each subband contains N^2 coefficients.

3. For each subband:

   i) Calculate the noise standard deviation [8].

   ii) Use the nonlinear mapping function defined by (2) to modify the subband coefficients.

4. Pass the modified coefficients through the synthesis part of the GDFB and reconstruct the enhanced image.

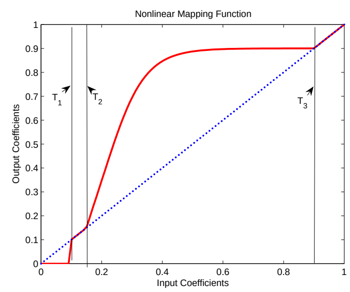

Figure 4: Enhancement map: $b = 0.20, c = 25, T_1 = 0.1, T_2 = 0.15, T_3 = 0.9$.

5. EXPERIMENTS

In this section, we compare the enhancement results obtained by the GDFB with those by the nonsubsampled wavelet transform (NSWT) using db4 wavelet and the NSCT. In this experiment, we used 1, 8, 8, 16, 16 directions in the scales from coarser to finer, respectively. We chose $b = 0.10, 0.11, 0.14, 0.15$ and $c = 5, 10, 30, 40$ for the directions in the scales from coarser to finer, respectively. The subband coefficients of the coarsest scale were not modified. From the experiments, we see that our new enhancement technique works better than that of the NSWT using our enhancement map and the NSCT as done in [4]. One of the major advantages of our algorithm compared to the NSCT is that it is very fast. Using an Intel 1.7 GHz Processor, it takes on average approximately 375 seconds for the NSCT algorithm to produce an enhanced image of size 256×256, whereas it takes only 25 seconds for the GDFB to enhance the same image running in MATLAB on a Windows XP system.

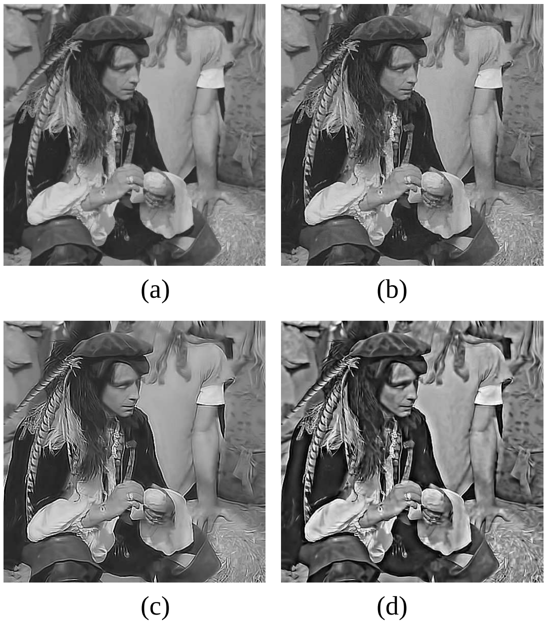

Figure 5: Image Enhancement experiment. (a) Original image. (b) Enhanced by the NSWT. (c) Enhanced by the NSCT. (d) Enhanced by the GDFB.

6. CONCLUSION

We presented new methods for creating an m-channel directional filter bank. The key component is the realization that the synthesis filters can be found by viewing the problem as a multichannel deconvolution problem. We used a shift invariant directional multiresolution image representation for image enhancement by nonlinear modification of coefficients in the subbands. We proposed a new enhancement algorithm that was capable of adaptively removing noise and enhancing salient features such as edges. Experimental results show that our enhancement technique achieves better results than the NSWT and the NSCT and is considerably faster than the NSCT.

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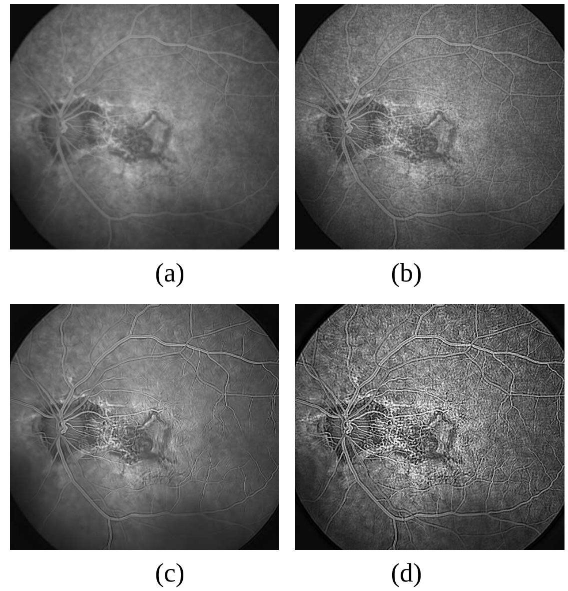

Figure 6: Image Enhancement experiment with an angiogram. (a) Original image. (b) Enhanced by the NSWT. (c) Enhanced by the NSCT. (d) Enhanced by the GDFB.

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