INTEGRABILITY-REGULARIZED PHASE UNWRAPPING VIA SPARSE ERROR CORRECTION

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ABSTRACT

We propose a new formulation of the classical two-dimensional phase unwrapping problem. Using a sparse-error, gradient-domain measurement model, we simultaneously seek the absolute phase and sparse gradient errors that minimize a novel energy functional that strongly encourages the integrability of the corrected gradient field. Our approach can be cast as a *generalized lasso* problem, and we compute the solution using the alternating direction method of multipliers (ADMM) algorithm. Adopting a commonly-used interferometric synthetic aperture radar noise model, we evaluate our technique for several synthetic surfaces.

Index Terms— Phase unwrapping, sparse error correction, integrability, interferometric synthetic aperture radar

1. INTRODUCTION

Phase unwrapping is a problem that arises in many applications, including magnetic resonance imaging, optical interferometry, and interferometric synthetic aperture radar (InSAR). The task is to infer the real-valued *absolute phase* from measurements of the *principal phase*. It is usually the case that the absolute phase carries the information of interest, but the principal phase is the only observable quantity. The two quantities are related as follows: if $\phi \in \mathbb{R}$ represents the absolute phase, then the corresponding principal phase value is given by $\psi = \mathcal{W}(\phi)$, where the wrapping operator $\mathcal{W}$ is defined as

$$\begin{aligned} &\mathcal{W} : \mathbb{R} \rightarrow [-\pi, \pi) \\ &\mathcal{W}(\phi) = [(\phi + \pi) \bmod 2\pi] - \pi, \end{aligned} \tag{1}$$

and applied componentwise in the case of multivariate $\boldsymbol{\phi}$. Using the above notation, the phase unwrapping problem is that of inferring $\boldsymbol{\phi}$ from $\boldsymbol{\psi}$.

Due to the many-to-one nature of $\mathcal{W}$, this problem is ill-posed. Therefore, in order to find a unique solution, additional constraints must be imposed on $\boldsymbol{\phi}$. One such constraint that is applied almost universally in the literature is derived from the *Itoh condition* [1]. Assuming a two-dimensional $\boldsymbol{\phi}$ that is comprised of phase samples obtained on a uniformly-spaced discrete grid, the Itoh condition is said to be satisfied if neighboring phase values do not differ by more than π. Whether or not this condition is satisfied is related to both the spatial sampling rate and the smoothness of the underlying physical quantity. However, if the Itoh condition is satisfied, then it can be shown that

$$\boldsymbol{\nabla}\boldsymbol{\phi} = \mathcal{W}(\boldsymbol{\nabla}\boldsymbol{\psi}), \tag{2}$$

where $\boldsymbol{\nabla}$ computes differences between four-connected neighbors. Unfortunately, even in the case of sufficiently-smooth $\boldsymbol{\phi}$ and sufficient spatial sampling, (2) may fail to hold due to noise in the system that acquires $\boldsymbol{\psi}$. Nevertheless, this gradient constraint is used in virtually every unwrapping procedure, and the distinguishing trait among these procedures is the way in which this issue is addressed.

In this paper, we propose a novel phase unwrapping technique that explicitly models the error in (2) as a *sparse* quantity, i.e., that significant inequality occurs in a relatively small number of locations. We find the error by explicitly enforcing integrability in the corrected gradient measurements. We formulate the unwrapping problem as one of jointly estimating both the absolute phase and the sparse errors, and cast it as a *generalized lasso* [2][3] problem. We then propose the use of the *alternating direction method of multipliers* (ADMM) algorithm [2] to compute the estimates.

1.1. Organization

This paper is organized as follows. In Section 2, we review related work in the field of phase unwrapping. In Section 3 we develop our formulation of sparse-error-corrected phase unwrapping, which culminates in an interpretation of the problem in the generalized lasso framework. In Section 4, we provide details of the optimization algorithm. We present the results of our technique in Section 5, and Section 6 concludes the paper with a summary.

2. RELATED WORK

Phase unwrapping is a problem that has received a great deal of attention from the research community. While early efforts focused on estimating the absolute phase directly from wrapped observations, more recent work has also dealt with the more limited task of denoising the wrapped observations (see, e.g., [4, 5]). These denoising methods do not explicitly perform the unwrapping, but they do often produce very good results when used to preprocess the input before applying absolute-phase-estimation techniques.

The method we present in this paper is one that computes an estimate of the absolute phase directly from wrapped, possibly noisy, observations. Virtually all such techniques rely on (2), which allows one to use $\boldsymbol{\psi}$ to generate measurements of the horizontal and vertical absolute phase differences. The unwrapping problem then becomes one of estimating a two-dimensional image ($\boldsymbol{\phi}$) from measurements of the corresponding gradient field. This more general problem is

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one with a wide variety of applications beyond that of phase unwrapping. For a thorough treatment, see [6].

Early attempts to solve the phase unwrapping problem included *path-following* [7, 8, 9] techniques, which seek suitable paths over which simple gradient-measurement integration can be used to compute the absolute phase. A more popular class of phase unwrapping techniques formulates the problem as one of energy minimization, i.e.,

$$\boldsymbol{\phi}^* = \arg\min_{\boldsymbol{\phi}} \mathcal{J}(\boldsymbol{\phi}), \tag{3}$$

where the distinguishing trait among these methods lies in how the energy function, $\mathcal{J}$, is defined. Hunt [10] selects

$$\mathcal{J}(\boldsymbol{\phi}) = \|\boldsymbol{\nabla}\boldsymbol{\phi} - \mathcal{W}(\boldsymbol{\nabla}\boldsymbol{\psi})\|_2^2, \tag{4}$$

where $\boldsymbol{\nabla}\boldsymbol{\phi} = \left[\boldsymbol{\nabla}_x\boldsymbol{\phi}^T \ \boldsymbol{\nabla}_y\boldsymbol{\phi}^T\right]^T$, and $\boldsymbol{\nabla}_x\boldsymbol{\phi}$ and $\boldsymbol{\nabla}_y\boldsymbol{\phi}$ denote the vectorized horizontal and vertical components, respectively, of the spatial gradient. Using this functional, $\boldsymbol{\phi}^*$ can be computed by solving the standard Poisson equation. Several modifications of (4) have been proposed [11], including the addition of regularization terms that encourage properties such as smoothness [12] and gradient integrability [13]. Others[14] generalize (4) to a p-norm. Certain values of p can affect the convexity of $\mathcal{J}$ and therefore require more creative computational techniques, such as approaches from network programming [15][16], in order to compute a solution.

The method we propose here is an energy-minimization approach to phase unwrapping. We select a $\mathcal{J}$ that enforces the gradient constraint as in (4) but is able to robustly handle outliers in $\mathcal{W}(\boldsymbol{\nabla}\boldsymbol{\psi})$ by simultaneously enforcing sparsity in an error term that ensures integrability. This formulation allows us to use a primal-dual algorithm in order to efficiently compute a solution.

3. PROBLEM FORMULATION

Let $\boldsymbol{\phi} \in \mathbb{R}^{mn}$ represent the unknown, vectorized, $m \times n$ absolute phase image. Similarly, let $\boldsymbol{\psi} \in [-\pi, \pi)^{mn}$ represent the corresponding wrapped observation. We shall assume that $\boldsymbol{\psi} = \mathcal{W}(\boldsymbol{\phi})$, where $\mathcal{W}$ is defined as in (1).

Let $\mathbf{G}_x \in \{-1, 0, 1\}^{mn \times mn}$ be a matrix that enables the computation of the vectorized forward-difference approximation to the horizontal component of the spatial gradient for an input image vectorized in column-major order, and let $\mathbf{G}_y \in \mathbb{R}^{mn \times mn}$ do the same for the vertical component. From these two matrices, we form $\mathbf{G} = \left[\mathbf{G}_x^T \ \mathbf{G}_y^T\right]^T$, a $2mn \times mn$ sparse matrix that we can use to computed the stacked, vectorized spatial gradient components, $\left[\boldsymbol{\nabla}_x\boldsymbol{\phi}^T \ \boldsymbol{\nabla}_y\boldsymbol{\phi}^T\right]^T$.

Let us also define a matrix we can use to compute the curl of a gradient field, which is usually done by considering two-by-two loop integrals over the underlying spatial domain. Let $\mathbf{p}, \mathbf{q} \in \mathbb{R}^{mn}$ define the horizontal and vertical components, respectively, of an $m \times n$ gradient field. Then the curl for a single two-by-two loop at spatial location (x, y) is defined as:

$$\begin{aligned}\texttt{curl}(y, x) =& p(y+1, x) - p(y, x) \\ &+ q(y, x) - q(y, x+1).\end{aligned} \tag{5}$$

where x and y denote the vertical and horizontal pixel coordinates, respectively. We can compute all mn curl values using a matrix-vector equation $\mathbf{C}\left[\mathbf{p}^T\mathbf{q}^T\right]^T$, where $\mathbf{C} \in \{-1, 0, 1\}^{mn \times 2mn}$ is defined such that each row computes (5) for a different spatial location.

Using the above, we now focus on our formulation of the phase unwrapping problem. Because the gradient constraint (2) is often violated, we model the error explicitly using

$$\mathbf{G}\boldsymbol{\phi} = \mathcal{W}(\mathbf{G}\boldsymbol{\psi}) - \mathbf{e}, \tag{6}$$

where $\mathbf{e} \in \mathbb{R}^{2mn}$ represents the error. Using this modified measurement model, we propose the following energy function for unwrapping:

$$\mathcal{J}_u(\boldsymbol{\phi}) = \|\mathbf{G}\boldsymbol{\phi} - (\mathcal{W}(\mathbf{G}\boldsymbol{\psi}) - \mathbf{e})\|_2^2 + |\mathbf{a}_k^T\boldsymbol{\phi}|^2, \tag{7}$$

where $\mathbf{a}_k$ is the k^{th} column of the identity matrix and the second term is included in order to address the unknown constant of integration.

While (7) enables the computation of an optimal $\boldsymbol{\phi}$, we have not yet addressed how to find $\mathbf{e}$. To this end, we examine the integrability of the measured gradient field. For noiseless gradient measurements, such as $\mathbf{G}\boldsymbol{\phi}$, (5) yields a value of zero for each loop. That is, the gradient field is *integrable* (also known as *irrotational* in the phase unwrapping literature [9]), or $\mathbf{C}\mathbf{G}\boldsymbol{\phi} = \mathbf{0}$. Left-multiplying both sides of (6) by $\mathbf{C}$ ultimately yields the following integrability constraint:

$$\mathbf{C}\mathbf{e} = \mathbf{C}\mathcal{W}(\mathbf{G}\boldsymbol{\psi}). \tag{8}$$

Especially in noise-free conditions, it is often the case that (2) is violated over a relatively small set of components, i.e., $\mathbf{e}$ is sparse. In a similar fashion to the work of Reddy *et al.* [17], we use the ℓ_1-norm as a proxy for sparsity and use $\|\mathbf{e}\|_1 = \sum_i |e_i|$ as a regularizer when seeking the optimal $\mathbf{e}$. We can also cast this problem as one of energy minimization with the functional

$$\mathcal{J}_e(\mathbf{e}) = \lambda_c\|\mathbf{C}\mathbf{e} - \mathbf{C}\mathcal{W}(\mathbf{G}\boldsymbol{\psi})\|_2^2 + \lambda_s\|\mathbf{e}\|_1, \tag{9}$$

where the values chosen for λ_c and λ_s specify the relative importance of satisfying each criterion.

Combining (7) and (9) above, our formulation can now be written as a single energy-minimization problem:

$$(\boldsymbol{\phi}^*, \mathbf{e}) = \arg\min_{\boldsymbol{\phi}, \mathbf{e}} \mathcal{J}_u(\boldsymbol{\phi}) + \mathcal{J}_e(\mathbf{e}) \quad . \tag{10}$$

3.1. Generalized Lasso Formulation

Optimization problem (10) can be rewritten as

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \lambda\|\mathbf{F}\mathbf{x}\|_1, \tag{11}$$

where $\lambda = \lambda_s$ controls the trade-off between satisfying the ℓ_2 and ℓ_1 constraints, and

$$\begin{aligned}\mathbf{x} &= \begin{bmatrix}\boldsymbol{\phi} \\ \mathbf{e}\end{bmatrix}, \\ \mathbf{A} &= \begin{bmatrix}\mathbf{G} & \mathbf{I} \\ \mathbf{a}_k^T & \mathbf{0}^T \\ \mathbf{0} & \lambda_c\mathbf{C}\end{bmatrix}, \\ \mathbf{F} &= \begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}, \\ \mathbf{b} &= \begin{bmatrix}\mathcal{W}(\mathbf{G}\boldsymbol{\psi}) \\ 0 \\ \lambda_c\mathbf{C}\mathcal{W}(\mathbf{G}\boldsymbol{\psi})\end{bmatrix}.\end{aligned} \tag{12}$$

The optimization problem (11) can be viewed as the *generalized lasso* problem [2][3] which can be efficiently solved via the alternating direction method of multipliers (ADMM) algorithm [2].

4. OPTIMIZATION

In a more general form, (11) can be seen as an instance of the following optimization problem

$$\min f(\mathbf{x}) + g(\mathbf{z}) \;\text{ such that }\; \mathbf{D}\mathbf{x} + \mathbf{H}\mathbf{z} = \mathbf{c}, \tag{13}$$

where f and g are convex functions. Rewriting the above with $f(\mathbf{x}) = \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2$, $g(\mathbf{z}) = \lambda\|\mathbf{z}\|_1$, $\mathbf{D} = \mathbf{F}$, $\mathbf{H} = -\mathbf{I}$, and $\mathbf{c} = \mathbf{0}$, the equivalence with (11) is made more clear:

$$\min \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \lambda\|\mathbf{z}\|_1 \;\text{ such that }\; \mathbf{F}\mathbf{x} = \mathbf{z}. \tag{14}$$

The ADMM method finds a saddle point for the augmented Lagrangian of the above, $\mathcal{F}_\rho$, by optimizing $\mathbf{x}$, $\mathbf{z}$, and Lagrange multipliers $\mathbf{u}$ sequentially (see [2] for details). Specifically, the iterations are defined as follows:

$$\mathbf{x}_{k+1} = \arg\min_{\mathbf{x}} \mathcal{F}_\rho(\mathbf{x}, \mathbf{z}_k, \mathbf{u}_k) \tag{15}$$

$$\mathbf{z}_{k+1} = \arg\min_{\mathbf{z}} \mathcal{F}_\rho(\mathbf{x}_k, \mathbf{z}, \mathbf{u}_k) \tag{16}$$

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \rho(\mathbf{F}\mathbf{x}_{k+1} - \mathbf{z}_{k+1}), \tag{17}$$

where the exact form of each iteration for our problem is given in Algorithm 1. For the $\mathbf{z}$-update step, $\mathcal{S}$ denotes the soft-thresholding operator, with the threshold given in the subscript. Since this method is the solution to our generalized lasso formulation of the phase unwrapping problem, we refer to this procedure as *phase unwrapping using the generalized lasso*, or PUGL.

Algorithm 1: PUGL

Input: λ,$\mathbf{b}$,$\mathbf{A}$,$\mathbf{F}$,ρ,maxIter
Initialization:
- Set Terminate $\leftarrow$ False.
- Set $\mathbf{z}_0 = \mathbf{0}, \mathbf{x}_0 = \mathbf{0}, \mathbf{u}_0 = \mathbf{0}$.
while (Terminate == False) **do**
- Calculate $\mathbf{x}_{k+1}$ by solving the following system of equations

$$(\mathbf{A}^T\mathbf{A} + \rho\mathbf{F}^T\mathbf{F})\mathbf{x}_{k+1} = (\mathbf{A}^T\mathbf{b} + \rho\mathbf{F}^T(\mathbf{z}_k - \mathbf{u}_k))$$

- Calculate $\mathbf{z}_{k+1}$ according to

$$\mathbf{z}_{k+1} = \mathcal{S}_{\frac{\lambda}{\rho}}\left(\mathbf{F}\mathbf{x}_{k+1} + \frac{\mathbf{u}_k}{\rho}\right)$$

- Calculate $\mathbf{u}_{k+1}$ according to

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \rho(\mathbf{F}\mathbf{x}_{k+1} - \mathbf{z}_{k+1})$$

- $k \leftarrow k + 1$
- **if** ($k \geq$ maxIter) **then**
  Terminate $\leftarrow$ True
end if
end while
Output: $\hat{\mathbf{x}} = \mathbf{x}_k$.

5. EXPERIMENTS

In this section, we shall describe the data, models, and experiments we used to evaluate the proposed unwrapping scheme.

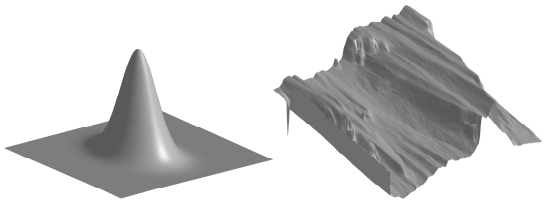

Fig. 1. Absolute phase surfaces used for experimental evaluation. Left: Gaussian surface. Right: Longs Peak.

5.1. Phase data

We tested our algorithm using two absolute phase surfaces. The first is a Gaussian surface that is synthetically generated: it is a 128×128 image, centered at $(0, 0)$, of a two-dimensional Gaussian with peak height 14π and $\sigma_x = 10$ and $\sigma_y = 15$. The second surface is the *Longs Peak* surface distributed with [9]: a real elevation map corresponding to a geographic area located in Colorado, USA. These surfaces are shown in Figure 1.

5.2. Noise model

In order to evaluate unwrapping performance in the presence of noise, we adopt a synthetic noisy observation model that is commonly used in InSAR phase unwrapping literature [18]. We form the noisy wrapped phase, $\boldsymbol{\psi}$ using

$$\psi = \arg\left(x_1 x_2^*\right), \tag{18}$$

where $x_1 = z_1 e^{j\phi}$, $x_2 = z_2$, and z_1 and z_2 are complex-valued random variables with $\mathbb{E}\left[|z_1|^2\right] = \mathbb{E}\left[|z_2|^2\right] = \theta^2$ and $\mathbb{E}\left[z_1 z_2^*\right] = \alpha\theta^2$. The parameter $\alpha \in [0, 1]$ is referred to as the *coherence*.

The level of noise in this observation model is determined by the value of α: $\alpha = 1$ indicates that there is no noise in ψ, while $\alpha = 0$ corresponds to a ψ that is comprised entirely of of noise. In order to evaluate unwrapping performance in the presence of noise, we performed experiments and generated results for every value of α in the set $\{0.7, 0.85, 1\}$.

Table 1. Surface Reconstruction MSE for Noisy Wrapped Phase Observations

Gaussian Surface	$\alpha = 0.70$	$\alpha = 0.85$	$\alpha = 1$
PhaseLa	72.18	16.84	1.30
PUMA	**5.09**	0.68	**0.00**
PUGL	6.58	**0.62**	0.00
Longs Peak	$\alpha = 0.70$	$\alpha = 0.85$	$\alpha = 1$
PhaseLa	476.00	408.01	334.50
PUMA	151.03	117.95	100.23
PUGL	**150.48**	**107.70**	**82.08**

5.3. Evaluation

The metric of evaluation we use is the mean-squared error between the true surface, $\boldsymbol{\phi}$, and the estimate $\hat{\boldsymbol{\phi}}$. In order to account for the unresolved degree of freedom that results from using only gradient measurements, we first ensure that each estimated surface has zero

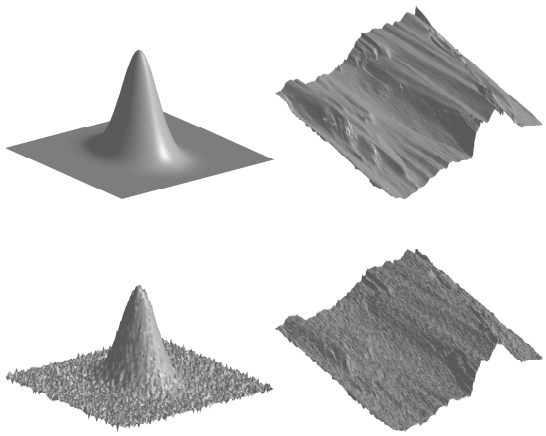

Fig. 2. PUGL-generated phase reconstructions. Left column: Guassian surface reconstructions. Right column: Longs Peak reconstructions. Top: reconstructions for $\alpha = 1$. Bottom: reconstructions for $\alpha = 0.85$.

mean, i.e., we calculate the mean-squared error according to

$$\text{MSE}\left(\boldsymbol{\phi}, \hat{\boldsymbol{\phi}}\right) = \frac{1}{mn} \sum_i \left[\left(\boldsymbol{\phi}_i - \bar{\boldsymbol{\phi}}_i\right) - \left(\hat{\boldsymbol{\phi}}_i - \bar{\hat{\boldsymbol{\phi}}}_i\right) \right]^2, \quad (19)$$

where $\bar{\boldsymbol{\phi}}$ and $\bar{\hat{\boldsymbol{\phi}}}$ denote the across-pixel mean values for $\boldsymbol{\phi}$ and $\hat{\boldsymbol{\phi}}$, respectively.

We compared our technique with two recent phase-unwrapping algorithms: PhaseLa [19] and PUMA [16]. To generate results, we used the implementations made available by the authors. For the PhaseLa algorithm, we used the ICI-adaptive approach with $H = [1, 2, 3, 4]$ and $\Gamma = 2.0$ (see [19] for definitions). For the PUMA algorithm, we used the convex clique potential induced by selecting $p = 2$.

Table 1 shows the mean-squared errors that result from each unwrapping procedure when using the noisy wrapped observations. It can be seen that PUGL usually performs better than the other algorithms presented, especially for the more-realistic Longs Peak surface. To visualize the unwrapping results, we show some of the PUGL-generated absolute phase estimates in Figure 2.

5.4. Parameters and error correction

To generate the PUGL estimates, parameter values $\lambda_c = 200$ and $\lambda_s = 1$ were used. We found that these values strongly enforced the integrability constraint while still allowing for $\mathbf{e}$ to be reasonably sparse. Figure 3 illustrates that $\mathbf{e}$ corrects wrapping artifacts: we show the estimated phase and significant components of the optimal $\mathbf{e}$ for a truncated version of the Gaussian surface (upper-left quadrant set to zero). The sharp discontinuity induced by truncation violates the Itoh condition, and it can be seen that this is exactly where the significant components of the optimal $\mathbf{e}$ cluster. The resulting phase estimate is a slightly-smoothed version of the true surface.

6. SUMMARY AND FUTURE WORK

In this paper, we proposed a novel formulation of the phase unwrapping problem, and provided a practical scheme by which to make the corresponding absolute phase estimate. We posed the problem as one of sparse error correction by explicitly modeling the error in gradient field measurements obtained from the wrapped phase. We then estimated the error term as one that induced an integrable gradient field while remaining as sparse as possible. We combined the above with a classical ℓ_2-based unwrapping scheme in such a way that the joint absolute phase and error estimation could be cast in the generalized lasso framework, and we used the ADMM algorithm to efficiently compute the optimal values. We termed the overall algorithm *phase unwrapping using the generalized lasso*, or PUGL, and examined its performance for a variety of surfaces and noise levels.

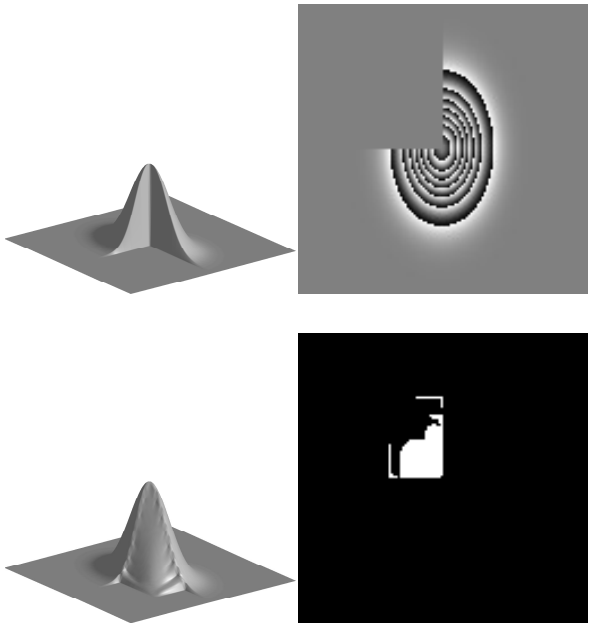

Fig. 3. Sparse error analysis. Upper-left: Truncated Gaussian surface. Upper-right: noise-free wrapped phase. Lower-left: PUGL estimate. Lower-right: locations of the significant components of the optimal $\mathbf{e}$.

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