This file contains the section B.1 “Predictive capabilities” of the document called “SAFIR. Technical documentation” and contained in the file named “Technical Reference of SAFIR.docx”.

PART 1: Exercises defined by the Belgium Ministry of Interior

These exercises have been defined by the department ArGEnCo of the University of Liege within a contract from the Belgium Ministry of Interior. They are defined in detail in the report [1]. They have been presented in a conference in Timisoara [2]. Most of these exercises have been taken from an original paper of Wickström & Palsson [3] who proposed and run them for validating the software TASEF [4]

EXERCISE 1

1. Objective

The aim of this exercise is to verify that the software correctly solves the Fourier's law of heat conduction when a section is discretised with regular quadrangular elements.

2. Exercise description

A square section discretised with quadrangular elements is considered. For symmetry reasons only a quarter of section is analysed (see Fig. 1). The thermal properties of the material do not depend on the temperature and the initial temperature of the section is 1000 °C. At t = 0 the section is plunged into medium, the temperature of which is and remains at 0 °C.

The thermal exchange on the surface is governed by the following equation:

$$q = h\,(T_g - T_s)$$

where q is the heat flux [W/m^2]; T_g is the temperature of the surrounding medium, i.e. 0 °C; T_s is the temperature of the surface [°C] and h is the convection coefficient [W/m^{2}K].

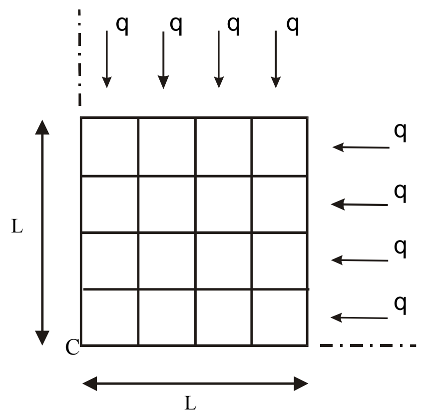

Figure 1. A quarter of the section discretised with 16 elements.

The reference solution at the centre of the whole square, i.e. at point C in Fig. 1, is given in Table 1 [5] for different values of the non-dimensional time.

Fo	T [°C]
0.0	1000.0
0.1	986.4
0.2	903.8
0.4	690.2
0.6	514.7
0.8	382.7
1.0	284.5

Table 1. Reference solution.

where *Fo* is the non-dimensional time defined as $Fo = \alpha\ t\ /\ L^2$, with α the thermal diffusivity of the material [m^2/s] and *t* the time [s].

Two models with different size and thermal properties, as reported in Table 2, were studied.

Model	λ	ρ	*C*	*L*	*h*
1	210	6	35	14	15
2	6	2	3	0.5	12

Table 2. Material thermal properties.

λ is the thermal conductivity [W/mK], ρ is the material density [kg/m^3]; *C* is the specific heat [J/kgK]; *L* is half of the square dimension [m] (see Fig. 1) and *h* is the convection factor [W/m^2K].

In order to check the influence of the mesh, both models were discretised with 4 meshes having 4, 16, 64 and 256 elements, respectively. Moreover, Model 1 discretised with 256 elements was analysed with different values of the time step in order to verify the influence of the time discretisation on the results: $\Delta t = Fo/10$, *Fo*/100 and *Fo*/1000.

3. Influence of the mesh: numerical results

The results of Model 1 and Model 2 presented in Tables 3 and 4 were obtained by using 10 000 time steps from Fo = 0 to Fo = 1.0[1]. They show for each mesh the temperature T and the difference Δ between the numerical and the analytical solution. Moreover, Fig. 2 and 3 show the evolution of the error for Fo = 0.1 and Fo = 1.0 as a function of the number of degrees of freedoms.

From Table 3, it is clear that the 4-element mesh is too coarse because at the beginning of the simulation the temperature in the central node increases instead of

[1] This is supposed to represent an infinitely short time step, in order to analyse only the effect of the geometrical discretisation.

decreasing[2]. It can be observed that the results in Tables 3 and 4 are exactly the same.

3.1 Model 1

	4 Elements		16 Elements		64 Elements		256 Elements	
Fo	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]
0.1	1021.0	34.6	994.0	7.6	988.1	1.7	986.7	0.3
0.2	937.1	33.3	912.0	8.2	905.8	2.0	904.2	0.4
0.4	707.3	17.1	694.7	4.5	691.6	1.4	690.8	0.6
0.6	524.2	9.5	517.4	2.7	515.7	1.0	515.2	0.5
0.8	388.1	5.4	384.4	1.7	383.5	0.8	383.3	0.6
1.0	287.3	2.8	285.6	1.1	285.2	0.7	285.1	0.6

Table 3. Model 1: comparison between the numerical and the analytical solution for different meshes.

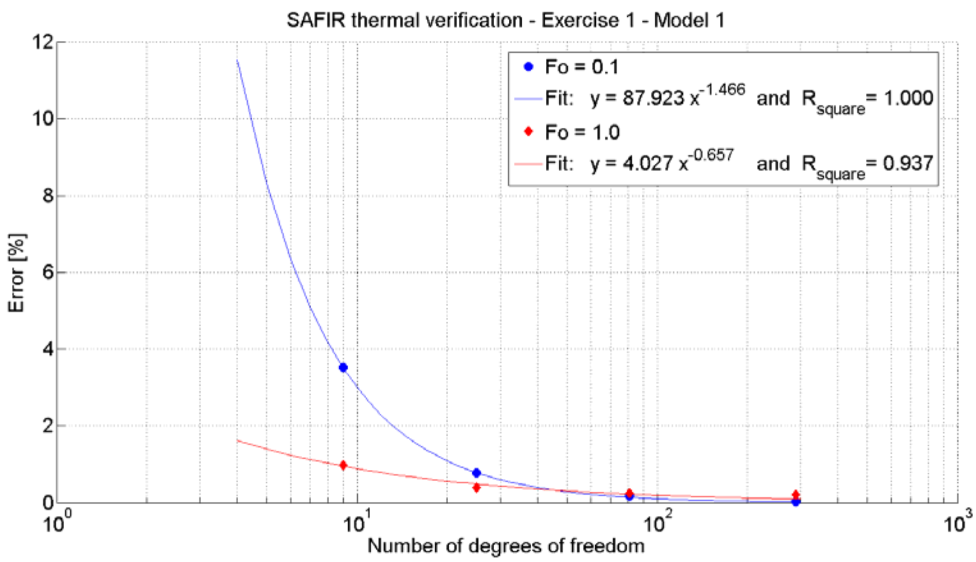

Figure 2. Model 1: error evolution for *Fo* = 0.1 and *Fo* = 1 as a function of the number of degrees of freedoms.

[2] This is due to the well known *skin effect* that appears in linear elements when too coarse a mesh is used.

3.2 Model 2

	4 Elements		16 Elements		64 Elements		256 Elements	
Fo	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]
0.1	1021.0	34.6	994.0	7.6	988.1	1.7	986.7	0.3
0.2	937.1	33.3	912.0	8.2	905.8	2.0	904.2	0.4
0.4	707.3	17.1	694.7	4.5	691.6	1.4	690.8	0.6
0.6	524.2	9.5	517.4	2.7	515.7	1.0	515.2	0.5
0.8	388.1	5.4	384.4	1.7	383.5	0.8	383.3	0.6
1.0	287.3	2.8	285.6	1.1	285.2	0.7	285.1	0.6

Table 4. Model 2: comparison between the numerical and the analytical solution for different mesh.

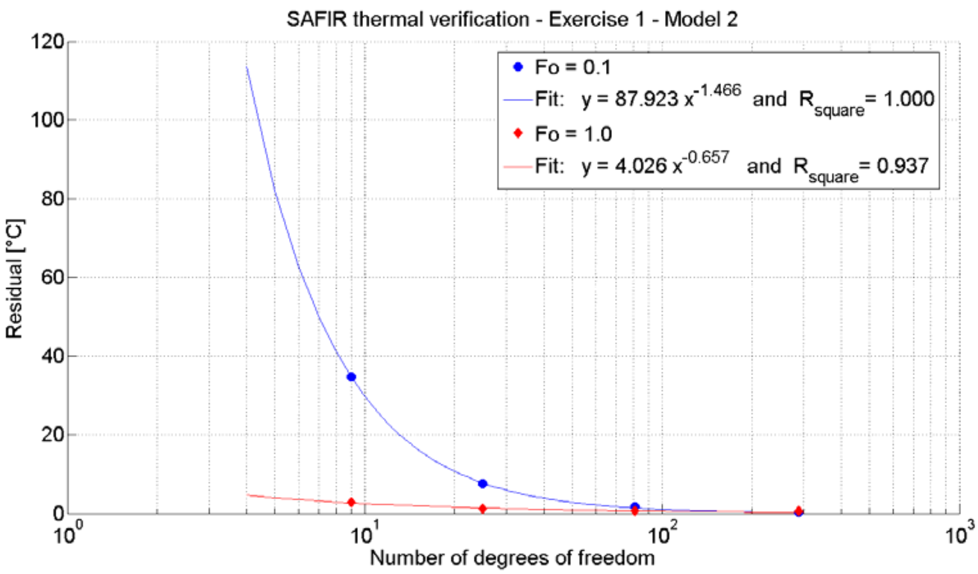

Figure 3. Model 2: error evolution for *Fo* = 0.1 and *Fo* = 1 as a function of the number of degrees of freedoms.

4. Influence of the time step: numerical results

Table 5 and Fig. 4 show the obtained results obtained for model 1 with a mesh of 256 elements, first the values, then in a graph.

	Δt = Fo / 10		Δt = Fo / 100		Δt = Fo / 1000	
Fo	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]
0.1	965.4	-21.0	983.3	-3.1	986.4	0.0
0.2	892.1	-11.7	902.6	-1.2	904.1	0.3
0.4	706.0	15.8	692.6	2.4	690.9	0.7
0.6	539.3	24.6	517.8	3.1	515.5	0.8
0.8	408.5	25.8	385.9	3.2	383.5	0.8
1.0	308.9	24.4	287.5	3.0	285.3	0.8

Table 5. Model 1: comparison between the numerical and the analytical solution for different time steps.

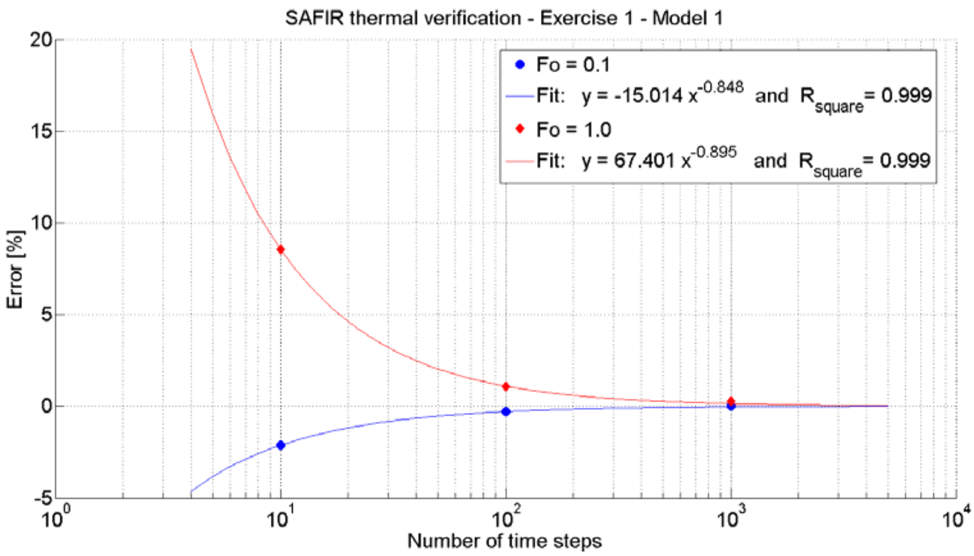

Figure 4. Model 1: error evolution for *Fo* = 0.1 and *Fo* = 1 as a function of the number of time steps when the section is discretised with 256 elements.

5. Conclusions

From the results it is possible to observe that the numerical solution converges to the analytical one, with an error that tends to zero, when the mesh is refined (the size of the elements is decreased) and when the time step tends toward zero.

EXERCISE 2

1. Objective

The aim of this exercise is to verify that the software correctly solves the Fourier's Law of Heat Conduction when a section is discretised with regular triangular elements.

2. Description of the exercise

A square section discretised with regular triangular elements according to two orientations is considered (Fig. 1). For symmetry reasons only a quarter of the section is analyzed. The thermal properties of the material do not depend on the temperature and the initial temperature of the section is 1000 °C. Then, at t = 0 the section is plunged into a medium, the temperature of which is and remains at 0 °C.

The thermal exchange on the surface is governed by the following equation:

$$q = h\,(T_g - T_s)$$

where q is the heat flux [W/m^2]; T_g is the temperature of the surrounding medium, i.e. 0 °C or 273 K; T_s is the temperature of the surface [°C or K] and h is the coefficient of convection [W/m^{2}K].

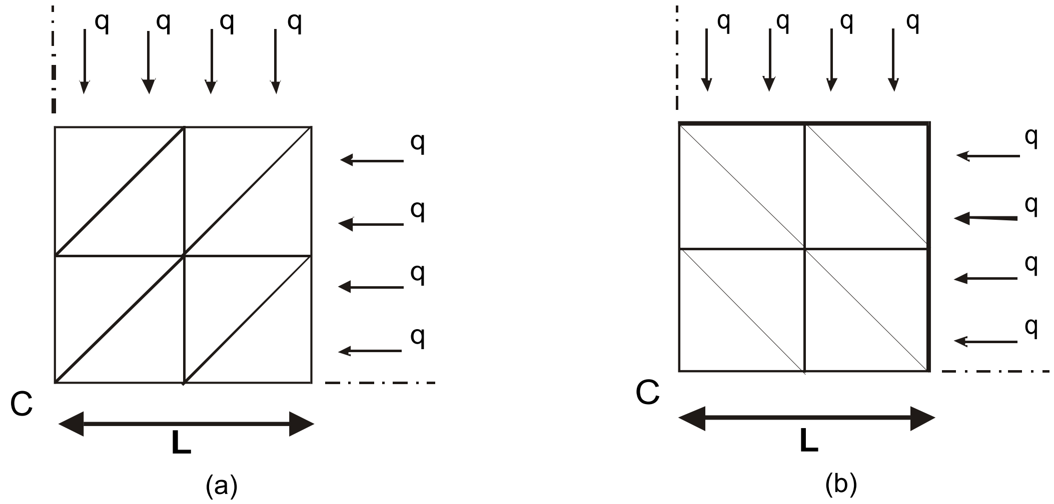

Figure 1. A quarter of the section discretised with 8 elements: a) model 1; b) model 2.

The reference solution at the centre of the section, i.e. at point C in Fig. 1, is given in Table 1 [2].

Fo	T [°C]
0.0	1000.0
0.1	986.4
0.2	903.8
0.4	690.2
0.6	514.7
0.8	382.7
1.0	284.5

Table 1. Reference solution.

where *Fo* is the non-dimensional time defined as $Fo = \alpha\ t\ /\ L^2$,α is the thermal diffusivity [m^2/s] and *t* the time [s].

The thermal properties are reported in Table 2.

λ	ρ	*C*	*L*	*h*
6	2	3	0.5	12

Table 2. Material thermal properties.

λ the thermal conductivity [W/mK], ρ is the material density [kg/m^3]; *C* is the specific heat [J/kgK]; *L* is the square dimension [m] and *h* is the convection factor [W/m^2K].

In order to check the influence of the mesh, both models with different orientation of the elements were discretised with 4 different meshes having 8, 32, 128 and 512 elements, respectively. Moreover, Model 1 discretised with 512 elements was analyzed with different values of the time step in order to verify this influence on the results: Δt = *Fo*/10, *Fo*/100 and *Fo*/1000.

3. Influence of the mesh: numerical results

The results of Model 1 and Model 2 presented in Tables 3 and 4 were obtained by using 10000 time steps from Fo = 0 to Fo = 1.0. They show for each mesh the temperature T and the difference Δ between the numerical and the analytical solution. Moreover, in Fig. 2 and 3 the error evolution - for Fo = 0.1 and Fo = 1.0 - as a function of the number of degrees of freedoms is shown, along with its fitting according to a y = a·xb equation and with the relative Rsquare statistical measure.

From Table 3, it is clear that the 8-element mesh is too coarse because at the beginning of the simulation the section warms up instead of cooling down because of skin effects inherent to linear finite elements used in a too coarse mesh. Also, comparing Tables 3 and 4, the results given by the second type of orientation are more accurate.

3.1 Model 1

	8 Elements		32 Elements		128 Elements		512 Elements	
Fo	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]
0.1	1038.4	52.0	999.2	12.8	989.6	3.2	987.1	0.7
0.2	955.6	51.8	919.9	16.1	908.4	4.6	905.0	1.2
0.4	722.4	32.2	701.4	11.2	693.8	3.6	691.5	1.3
0.6	536.3	21.6	522.5	7.8	517.4	2.7	515.8	1.1
0.8	397.7	15.0	388.3	5.6	384.8	2.1	383.7	1.0
1.0	294.9	10.4	288.6	4.1	286.2	1.7	285.4	0.9

Table 3. Model 1: comparison between the numerical and the analytical solution for different mesh.

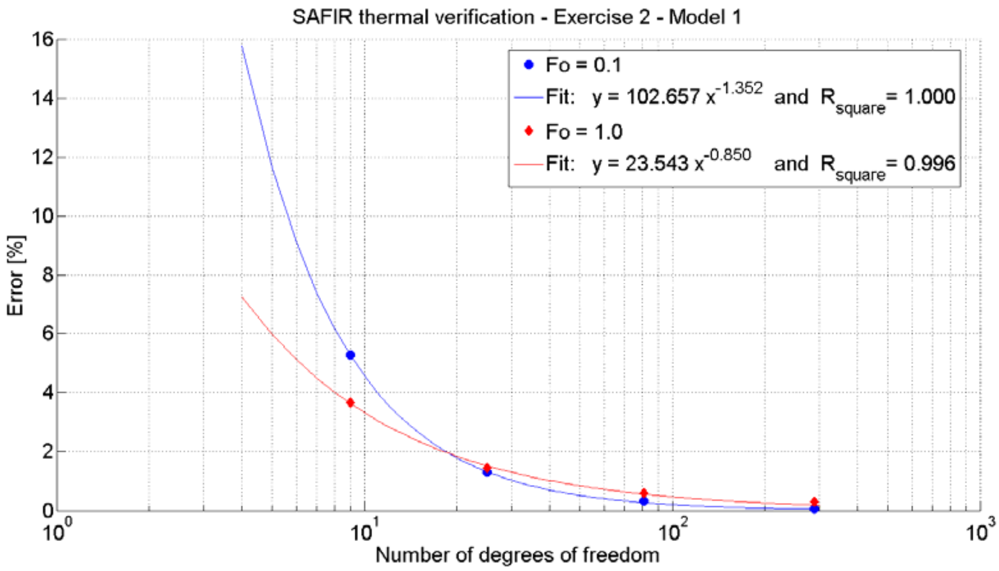

Figure 2. Model :1 evolution of the error for *Fo* = 0.1 and *Fo* = 1 as a function of the number of degrees of freedoms

3.2 Model 2

	8 Elements		32 Elements		128 Elements		512 Elements	
Fo	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]
0.1	995.7	9.3	988.0	1.6	986.5	0.1	986.2	-0.2
0.2	905.9	2.1	902.2	-1.6	902.8	-1.0	903.3	-0.5
0.4	682.1	-8.1	685.9	-4.3	688.8	-1.4	689.9	-0.3
0.6	505.1	-9.6	510.5	-4.2	513.5	-1.2	514.6	-0.1
0.8	373.6	-9.1	379.2	-3.5	381.9	-0.8	382.8	0.1
1.0	276.3	-8.2	281.6	-2.9	283.9	-0.6	284.7	0.2

Table 4. Model 2: comparison between the numerical and the analytical solution for different meshes.

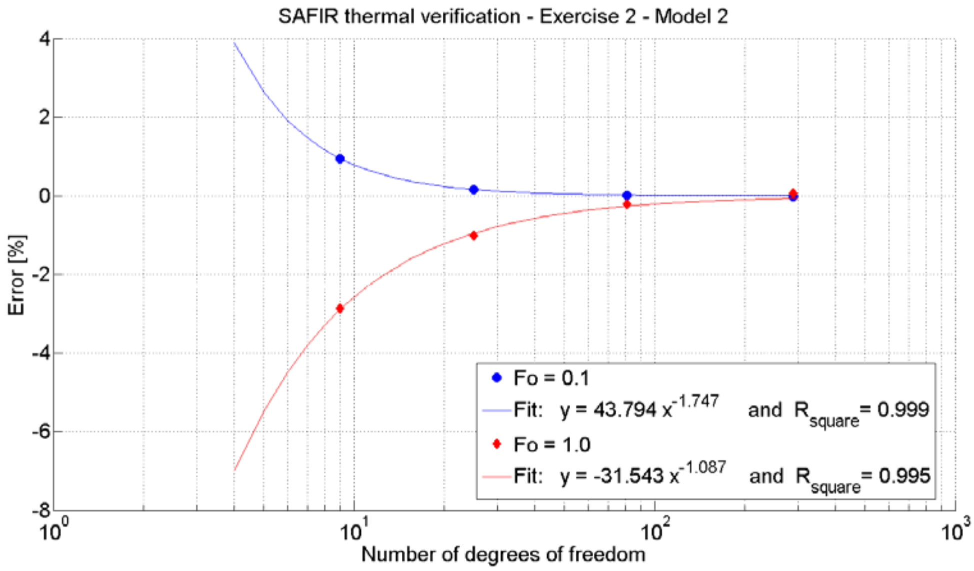

Figure 3. Model 2: evolution of the error for *Fo* = 0.1 and *Fo* = 1 as a function of the number of degrees of freedoms.

4. Influence of the time step: numerical results

Analogous result presentation as for Section 3 is shown in Table 5 and Fig. 4.

	Δt = Fo / 10		Δt = Fo / 100		Δt = Fo / 1000	
Fo	T [°C]	Δ [°C]	T [°C]	Δ [°C]	T [°C]	Δ [°C]
0.1	965.7	-20.7	983.7	-2.7	986.8	0.4
0.2	892.7	-11.1	903.3	-0.5	904.9	1.1
0.4	706.7	16.5	693.3	3.1	691.6	1.4
0.6	539.8	25.1	518.4	3.7	516.0	1.3
0.8	408.9	26.2	386.3	3.6	383.9	1.2
1.0	309.2	24.7	287.8	3.3	285.6	1.1

Table 5. Model 1: comparison between the numerical and the analytical solution for different time steps.

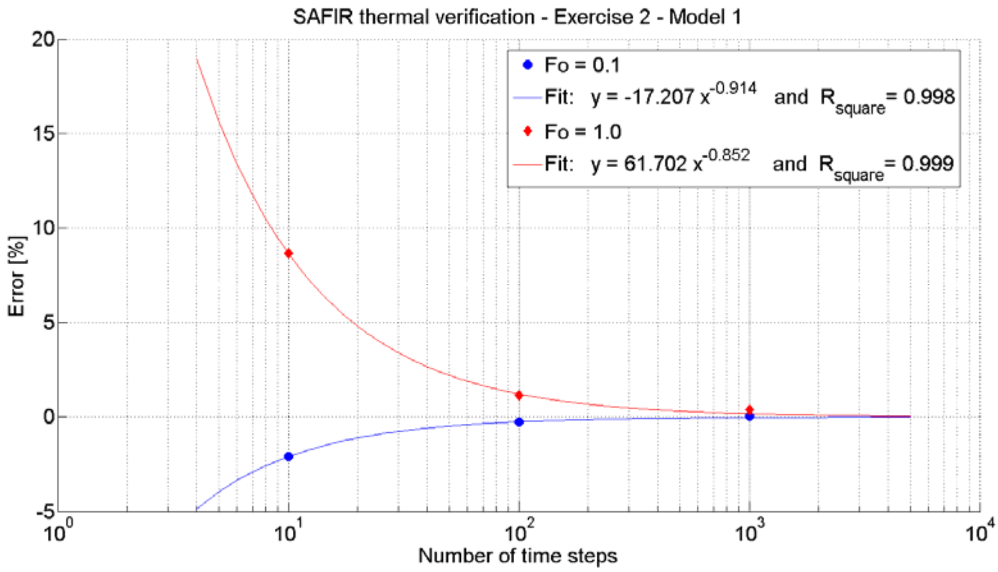

Figure 4. Model 1: evolution of the error for *Fo* = 0.1 and *Fo* = 1 as a function of the number of time steps when the section is discretised with 512 elements.

5. Conclusions

From the results it is possible to observe that the numerical solution converges to the analytical one, with an error that tends to zero, when the mesh is refined (the size of the elements is decreased) and when the time step tends toward zero.

6. References

[1] *Série de cas tests pour validation de programmes numériques modélisant le comportement des structures en cas d'incendie*. Ministère de l'intérieur Belge–Université de Liège (2000).

[2] *Evaluation of the thermal part of the code SAFIR by comparison with the code TASEF,* D. Pintea & J.-M. Franssen, Proceedings of the 8th International Conference on Steel Structures, Vol. 2, M. Ivan ed., MIRTON, Timisoara, (1997), 636-643, http://hdl.handle.net/2268/36226

[3] *An evaluation scheme of computer codes for calculating temperature in fire exposed structures - two dimensional analysis*, Wickström, U. & Palsson, J, SP Swedish National Testing and Research Institute, Fire Technology, SP Report 1999:36, Boras, Sweden (1999)

[4] TASEF - Temperature analysis of Structures Exposed to Fire - User's manual, Sterner, E & Wickström, U, J, SP Swedish National Testing and Research Institute, Fire Technology, SP Report 1990:05, Boras (1990)

[5] *Heat conduction in solids*, Carslaw & Jaeger, Oxford University Press, 2nd edition (1962).

PART 2: