$2D/3C$ MODEL OF LARGE SCALE STRUCTURES IN TURBULENT PLANE COUETTE FLOW

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ABSTRACT

A streamwise constant projection of the Navier Stokes equations forced by small-amplitude Gaussian noise is simulated. It is found that this model captures salient features of fully developed turbulent plane Couette flow. In particular, the nonlinearity in the model captures the mathematical mechanism that results in the characteristic shape of the turbulent velocity profile. Further, using Taylor's hypothesis the model also generates large scale streaky structures, that closely resemble large scale features that have been observed in DNS and experiments.

INTRODUCTION

Even though there are many open questions about turbulence in wall bounded shear flows, there has been a lot of progress in understanding some of its essential aspects. For example, the shape of the turbulent mean velocity profile is well known. In addition, there is a growing body of work that supports the notion that the flow field is characterized by coherent structures. In particular, the prevalence and importance of long streamwise structures has been underscored through the discovery of both near wall streaks (Kline et al., 1967) and, more recently, very large scale motions in high Reynolds number experiments (for example, Kim and Adrian, 1999; Morrison et al., 2004; Hutchins and Marusic, 2007). Near wall streaks have been well-studied, while large scale streamwise (and quasi-streamwise) features in the core are less well understood.

Large scale streamwise motions (i.e., structures having long streamwise correlations in the outer layer) are thought to be somewhat similar in nature to the near wall structures, but longer in extent. Features with streamwise extent $O(\delta)$ have been associated with the hairpin packet paradigm while motions of $O(10\delta)$ were observed in pipes by Kim and Adrian (1999) and Morrison et al. (2004). These very large scale motions consist of long coherent regions of low and high momentum, relative to the mean, (which we call stripes to distinguish them from the near wall streaks). Coherent structures that persist with even greater streamwise length, (up to 25 pipe radii or channel half lengths), as well as a large wall normal range, (with some azithmuthal variation) were more recently observed in pipes and channel flows (Monty et al., 2007). Streamwise elongated structures that meander, (i.e. slowly vary across the span over streamwise distance) with an extent $> 20\delta$ have also been identified in boundary layers (Hutchins and Marusic, 2007).

Analysis of the input-output response of the linearized Navier Stokes (LNS) equations has also concluded that streamwise constant features are the dominant mode shapes that develop under various perturbations about both the laminar (Jovanović and Bamieh, 2005) and turbulent mean velocity (del Álamo and Jiménez, 2006) profiles. The perturbed LNS equations are thought to be adequate to capture the energy production of the full nonlinear system because it is widely believed that the nonlinear terms are passive (energy conserving), thus energy can only be amplified through linear interactions (Trefethen et al., 1993). In this paradigm Butler and Farrell (1992) were able to conclude that streaks of streamwise velocity naturally arise from the set of initial

conditions that produce the largest energy growth in wall bounded shear flows and that the disturbances associated with this maximum amplification are streamwise vortices. In wall bounded flows, this alignment of structures in the streamwise configuration is also consistent with the structures that are most amplified under random disturbances of the linearized evolution equations, see for example (Farrell and Ioannou, 1993).

Although linear models have proven useful in studying the dominant mode shapes and the linear mechanisms associated with amplification of these modes, they are unable to capture the full picture. For example, they are unable to reproduce the change in the mean velocity profile as the flow transitions from laminar to turbulent. Further, linear analysis can only give local information regarding the full nonlinear system. Unfortunately, the full nonlinear Navier Stokes (NS) equations are analytically intractable.

The model presented herein represents a step toward understanding the full nonlinear picture. It is developed based on the assumption that fully developed turbulent flow is streamwise constant (i.e. does not vary in the streamwise direction). This assumption is mathematically realized by setting all of the streamwise velocity derivatives in the full NS equations to zero. Given that the shape of the mean velocity profile is an important flow feature, one would also like to have a model that can reproduce it. For this reason the model maintains the nonlinearity responsible for developing the well-known shape of the mean velocity profile. The idea that a streamwise constant model is sufficient to capture mean profile changes from laminar to turbulent is strongly supported by the work of Reddy and Ioannou (2000) where they show that the nonlinear interactions between the $[0, \pm 1]$ modes are the primary factor in determining the turbulent mean velocity profile. In the present work this so-called $2D/3C$ model for plane Couette flow is simulated under small amplitude Gaussian forcing and the resulting mean profile is compared to full DNS data and experiments.

Another goal of this work is to take a step toward understanding the full impact of streamwise constant features on the flow. In particular, the extent to which the streamwise component of the flow combined with the application of Taylor's hypothesis at the centerline can be used to reconstruct information about the upstream velocity field in the core.

The core region of Couette flow is of particular interest because of observations of a noticeable peak in the Fourier energy spectrum of the turbulence intensity at low frequencies (Komminaho et al., 1996; Kitoh and Umeki, 2008). In addition, structures reminiscent of stripes have long been observed in the core through DNS of turbulent plane Couette flow (Lee and Kim, 1991; Bech et al., 1995). These so-called 'roll cells' consist of persistent counter-rotating streamwise vortices at channel center. More recent DNS done at higher resolution and with longer box sizes (Komminaho et al., 1996; Tsukahara et al., 2006) have shown long streamwise alternating high and low speed streaky structures at the centerline that are more similar in character to stripes. The Couette flow experiments of Tillmark and Alfredsson (1998) found further evidence of 'stripe like' meandering structures in the form of long autocorrelations $R_{uu}(\tau)$ or two point correlations $R_{uu}(\Delta x)$ as well as periodic variation of spanwise correlations $R_{uu}(\Delta z)$ in the core. The streamwise extent of these correlations was longer than those generally seen in other wall bounded flows. Komminaho et al. (1996) also found that in contrast to other flows, streamwise correlations for Couette flow are larger at the center than near the wall. At channel center the zero cross distances of $R_{uu}(\tau)$ and

$R_{uu}(\Delta x)$ have been observed to be 3 times that of the corresponding structure in Poiseuille flow (Kitoh et al., 2005).

Although there is a great deal of evidence to support the existence of these long 'stripe like' structures, the inability to separate them from small scale turbulent motions that persist throughout the flow has made them difficult to characterize. Hamilton et al. (1995) attempted to isolate near wall streaky structures by performing DNS of a highly constrained or 'minimal Couette flow', where the box size was limited to approximately the minimum value required to capture the average spanwise spacing of a streak and maintain turbulent activity. Their method was to start with a fully developed flow and then continue the simulation with this minimal box size. They were also able to capture long streaks in the core, however due to the limited box size they were unable to elucidate their full extent. Komminaho et al. (1996) attempted to decouple the large streamwise structures from small scale phenomenon through the application of a local Gaussian filter to the streamwise (u) velocity fluctuations at the centerline. Using this technique they were able to identify streamwise elongated vortex-streak structures that were not fixed in either space or time. Experiments aimed at recreating a type of 'minimal Couette flow' were carried out by Kitoh and Umeki (2008) through the use of Vortex Generators. This methodology enabled the authors to filter out some of the small scale turbulent motions and study large scale streaky structures at the centerline.

The present work describes the output response of the $2D/3C$ model to a small amplitude noise forcing. The results illustrate the ability of this model to capture some aspects of the flow statistics as well as the large scale streaky structures that have been observed in both experiments and numerical studies. In particular, they demonstrate that the nonlinearity in the $2D/3C$ model is in fact the nonlinearity required to capture the turbulent mean velocity profile. This paper is organized as follows; the next section describes the model and simulation method. This is followed by a comparison of the simulation results to DNS data and experiments as well as some concluding remarks.

DESCRIPTION OF MODEL AND SIMULATION

The $2D/3C$ Model

The $2D/3C$ model discussed herein is comprised of two equations; one in terms of the spanwise/wall normal stream function $\psi(y, z, t)$, and the other in terms of the streamwise velocity $u'_{sw}(y, z, t)$. The velocity field is decomposed such that $\mathbf{u}(y, z, t) = \mathbf{U} + \mathbf{u}'_{sw}$, where $\mathbf{U} = (U(y), 0, 0)$ is the laminar profile and $\mathbf{u}'_{sw} = (u'_{sw}, v'_{sw}, w'_{sw})$ are the streamwise constant deviations from laminar. The Reynolds number employed is $Re_w = \frac{U_w h}{\nu}$, where U_w is the velocity of the top plate, $h = 2\delta$ is the channel height and ν is the kinematic viscosity of the fluid. The lower plate is stationary and all of the variables are non-dimensionalized with respect to U_w and h.

The stream function which forces the resulting model to satisfy the appropriate two dimensional continuity equation is given by $v'_{sw} = \frac{\partial \psi}{\partial z}$; $w'_{sw} = -\frac{\partial \psi}{\partial y}$. The full model, whose derivation is fully described in Bobba (2004), is

$$
\begin{aligned}
\frac{\partial u'_{sw}}{\partial t} &= -\frac{\partial \psi}{\partial z}\frac{\partial}{\partial y}\left(u'_{sw} + U\right) + \frac{\partial \psi}{\partial y}\frac{\partial u'_{sw}}{\partial z} + \frac{1}{Re}\Delta u'_{sw} \\
\frac{\partial \Delta \psi}{\partial t} &= -\frac{\partial \psi}{\partial z}\frac{\partial \Delta \psi}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial \Delta \psi}{\partial z} + \frac{1}{Re}\Delta^2 \psi
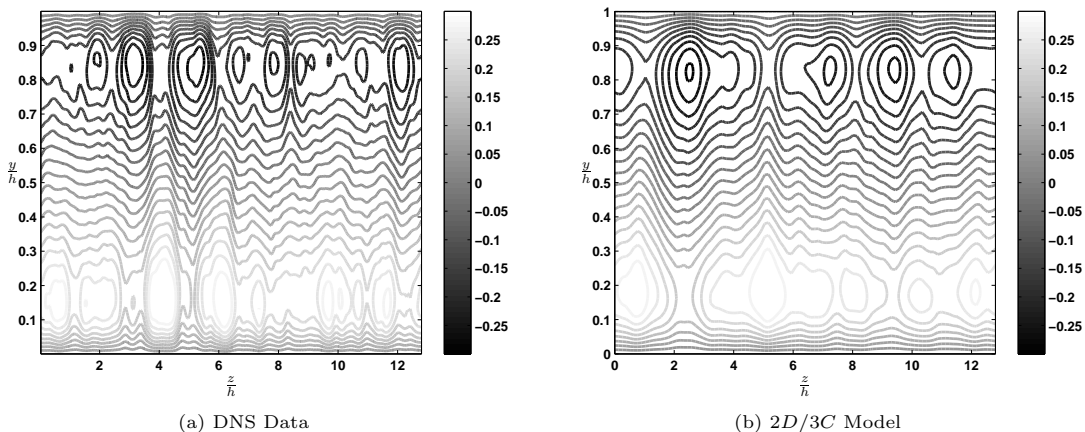\end{aligned}
\tag{1}
$$

(a) DNS Data   (b) $2D/3C$ Model

Figure 1: Contour Plots of (a) $u'_{x_{ave}}(y,z)$ from DNS data (b) $\overline{u'_{sw}(y,z,t)}$ from the $2D/3C$ model with $\sigma_{noise} = 0.01$, both at $Re_w = 3000$.

This model for plane Couette flow is more tractable than the full NS equations yet it captures many of the important flow features lost in a two dimensional model by maintaining all three velocity components. The laminar flow solution of this unforced model was previously shown to be globally, that is nonlinearly, stable for all Reynolds numbers (Bobba, 2004), so without forcing the response decays back to laminar flow. It is also an extension over linear models because it is the nonlinearity in the $u'_{sw}(y,z,t)$ equation that provides the mathematical mechanism for the redistribution of the fluid momentum that results in larger streamwise velocity gradients in the wall normal direction and gives the turbulent profile its characteristic S-shape.

As with any model, there are assumptions built into the $2D/3C$ model, and it is important to understand how these relate to the physical phenomena associated with turbulent flows. First, by thinking of the flow as streamwise constant in the 'mean', the smallest scale activity is lost. The streamwise constant assumption also specifically eliminates dynamics associated with hairpin-like eddies, which are thought to be related to both burst and sweep events. Although this limits our analysis, for example it makes appropriate scaling relationships more difficult to determine, it allows us to characterize turbulent structures that are formed in the absence of small scale effects.

Simulation

The nonlinearity in the $u'_{sw}(y,z,t)$ equation is necessary to capture the mechanism which allows cross-stream velocity components to drive deviations from the laminar mean velocity profile. However, the $\Delta\psi(y,z,t)$ equation can be linearized. This simplification makes sense in terms of the physics of the problem because the advection terms play a lesser role in the formation of the streaks and vortices that are important in determining the mean flow statistics. Mathematically, given that the $\Delta\psi(y,z,t)$ equation is driven by small amplitude noise, both $\psi(y,z,t)$ and $\Delta\psi(y,z,t)$ will also have a small amplitude and therefore the nonlinearities will have a negligible effect. Thus, the model governing the numerical studies described herein is given by;

$$\frac{\partial u'_{sw}}{\partial t} = -\frac{\partial \psi}{\partial z}\frac{\partial}{\partial y}\left(u'_{sw} + U\right) + \frac{\partial \psi}{\partial y}\frac{\partial u'_{sw}}{\partial z} + \frac{1}{Re}\Delta u'_{sw} + d_u$$

$$\frac{\partial \Delta\psi}{\partial t} = \frac{1}{Re}\Delta^2\psi + d_\psi \qquad (2)$$

where $d_u(y,z,t)$ and $d_\psi(y,z,t)$ are small amplitude Gaussian noise forcing (perturbations) with the amplitude defined in terms of the standard deviation which is denoted σ_{noise}. Stochastic forcing applied to the LNS equations is believed to be a plausible model of realistic flow-field forcing, as it has been previously shown to lead to flows that are dominated by streamwise elongated streaks and vortices (Farrell and Ioannou, 1993) that are strikingly similar to those observed in experiments.

The second equation is then just the stochastically forced heat equation in terms of $\Delta\psi(y,z,t)$. This equation is a linear stochastic partial differential equation which can be solved analytically. In the present work, however this is not pursued as a simulation is a much simpler way to demonstrate the efficacy of the model and an exposition on Itô calculus and Wiener chaos expansions is beyond the scope of this paper.

Any unmodeled phenomena such as the disturbances that may be amplified in an experiment or simulation as well as any missing or incorrect model parameters can be thought of as model uncertainties and mathematically represented as external forces applied to the system. These uncertainties may represent physical conditions that are difficult to characterize such as wall roughness or wall vibration as well as effects that can be characterized by adding additional complexity to the model such as thermal fluctuations, acoustic noise, or any other unmodeled conditions that tend to be present in experiments or numerical simulations. For example, in DNS and LES the disturbances or uncertainties may arise from the build up of numerical error. Other unmodeled effects that may be represented in this way could include the $k_x \neq 0$ modes that are not accounted for in the $2D/3C$ model. Of course the mean velocity profile is intrinsically linked to the full three dimensional flow-field through the mean streamwise momentum equation. One can then think of the main driving factor in the transition to turbulence as lack of robustness. The underlying equations are not robustly stable to disturbances and/or other uncertainties and thus, the performance of the system degrades.

In the present work we set $d_u = 0$ in equation (2) and approximate the uncertainties, $d_\psi(z,y,t)$, as zero mean small amplitude Gaussian noise forcing that is evenly applied at each grid point in the $y - z$ plane. This results in one-way coupling between the equations and in effect ψ can be seen as forcing for the u'_{sw} equation. This form of input noise represents forcing v'_{sw} and w'_{sw} and studying the response

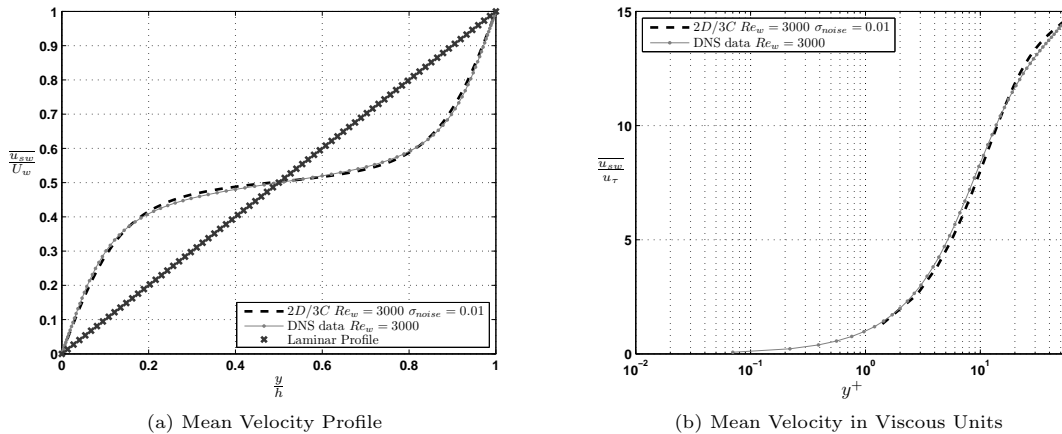(a) Mean Velocity Profile

(b) Mean Velocity in Viscous Units

Figure 2: (a) $\overline{u_{sw}(y,z,t)}$ from $2D/3C$ Model at $\sigma_{noise} = 0.01$ plotted with $\overline{u(x,y,z,t)}$ from DNS, (b) u^+ versus y^+ from $2D/3C$ Model at $\sigma_{noise} = 0.01$ and DNS Data both based on $G = 0.1991$, $Re_\tau = 52$.

in u'_{sw}. We chose this forcing configuration based on previous studies (Jovanović and Bamieh, 2005) that have shown that body forcing in the streamwise direction has a much smaller effect on the velocity field than forcing applied in the spanwise-wall normal $(y - z)$ plane. That work also demonstrated that the response to forcing in the $y - z$ plane is most strongly seen in the streamwise velocity fluctuations, as expected due to mean flow anisotropy.

In the present work results from simulations at two different Reynolds number are described. Case 1 is at $Re_w = \frac{U_w h}{\nu} = 3000$ and the forcing, d_ψ, is zero mean with standard deviation 0.01, (i.e. noise amplitude $= \sigma_{noise} = 0.01$). The computational box is $L_y \times L_z = h \times 12.8h$ with 75×100 grid points; the spanwise extent of $12.8h$ was selected to provide a direct comparison to the DNS data from Tsukahara et al. (2006). The time window used for computing time averages is $\Delta t = 100000 \frac{h}{U_w}$. Case 2 is at $Re_w = 12800$, $\sigma_{noise} = 0.004$, with a computational box of $L_y \times L_z = h \times 16.6h$ with 75×130 grid points. All simulations described herein were carried out using a basic second order central difference scheme in both the spanwise (z) and wall-normal (y) directions with periodic boundary conditions in z and no-slip boundary conditions in y.

RESULTS AND DISCUSSION

In the present work simulation results from the $2D/3C$ model are compared with full field DNS data which we obtained from the Kawamura group (Tsukahara et al., 2006). For this data, $Re_w = 3000$. To compare the flow features that arise from simulation of the $2D/3C$ model we took a streamwise (x) average of the data. In the following discussion the x-average of the full velocity field of DNS is denoted, $\mathbf{u}_{x_{ave}} = (u'_{x_{ave}} + U(y), v'_{x_{ave}}, w'_{x_{ave}})$, while time averages are indicated by an overbar, $\overline{(\cdot)}$.

Comparison of the contour plots of $u'_{x_{ave}}(y,z)$ from the DNS data and $\overline{u'_{sw}(x,y,t)}$ from the $2D/3C$ simulation, (pictured in Figures 1(a) and 1(b) respectively) shows that contours of constant $\overline{u'_{sw}}$ agree well with those of $u'_{x_{ave}}$ from the DNS. These plots also show that the offset in spatial phase from top to bottom between the maximum magnitude of deviations from laminar that is seen in the DNS data and observed in experiments is also reproduced through simulation of equations (2).

A fast Fourier transform over the span of $u'_{x_{ave}}$ and

analysis using the techniques of Farrell and Ioannou (1993) estimates the z wavelength of the DNS data to be roughly $k_z = 1.84$. Frequency analysis of $\overline{u'_{sw}}$ from the $2D/3C$ model indicates that most of the energy from the $2D/3C$ simulation is approximately $4 \le k_z \le 6.1$, however it is similarly clear in Figure 1(b) that there is also a significant contribution from $k_z \approx 2$.

Mean Velocity Profile

Figure 2(a) shows that the mean velocity profile $\overline{u_{sw}(y,z,t)}$ for Case 1, $Re_w = 3000$, shows good agreement with the DNS at the same Reynolds number.

The definition of friction coefficient (C_f) for Couette flow is

$$C_f = \frac{\tau_w}{\frac{1}{2}\,\rho\,(\frac{1}{2}U_w)^2} \qquad (3)$$

where τ_w is the shear stress at the wall. Using equation (3) along with the following relationship for the skin friction proposed by Robertson (1959)

$$\sqrt{\frac{C_f}{2}} = \frac{G}{\log_{10}(\frac{1}{4}Re_w)} \qquad (4)$$

and the empirical constant G we computed the friction velocity u_τ and friction Reynolds number Re_τ.

Figure 2(b) shows the same velocity profiles as in Figure 2(a) in inner units using the value $G = 0.1991$ from Tsukahara et al. (2006) which corresponds to $Re_\tau = 52$. The overall agreement of the two curves is good, although it is clear that below $y^+ \approx 20$ the $2D/3C$ model is underestimating the expected velocity profile, (max error 7.4%) and above that it is overshooting it, (max error 2.4%). The noise is modeled as being evenly distributed across the grid while in reality the noise is likely higher in the buffer region and lower in the overlap layer. Further study of the model with different noise distributions may improve the agreement.

Centerline Streak Pattern

While the $2D/3C$ model provides us with a $y - z$ snapshot in space, in general it is unclear how to reconstruct the the streamwise information. The model is designed to capture the mean features of the flow and we are particularly interested in understanding the spatial distribution of large scale features associated with the model. The simplest way to reconstruct the streamwise information is to convect the
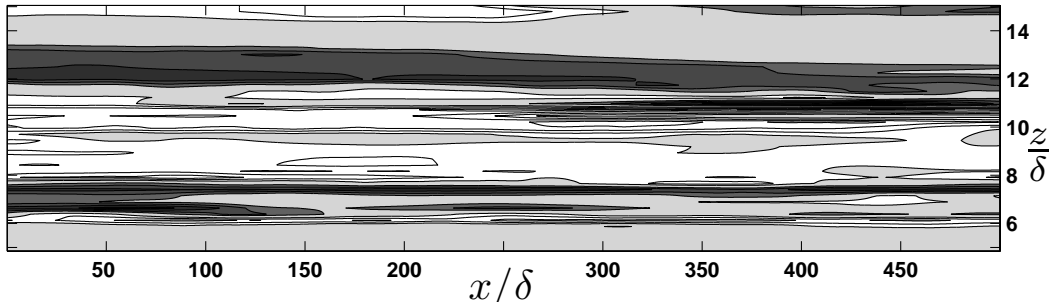
Figure 3: Typical streak pattern on the central plane of fully developed turbulent plane Couette flow from the $2D/3C$ model at $Re_w = 12800$ ($Re_c = 3200$). Dark regions are low speed streaks, $\frac{|u'_{sw}|}{|U_c|} \leq -0.05$, and the white regions are high speed streaks, $\frac{|u'_{sw}|}{|U_c|} \geq 0.05$, light grey regions are regions without streaks.
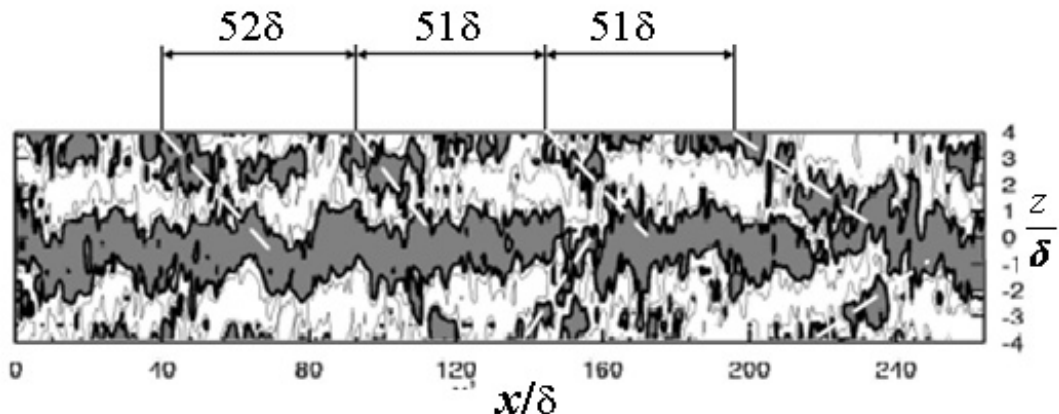

Figure 4: Figure 15 from Kitoh and Umeki, (2008) relabeled to match flow variables from the present work. Typical streak pattern on the central plane of Couette flow with Vortex Generators at $Re_w = 15000$, ($Re_c = 3750$). Dark regions $\frac{|u'_{sw}|}{|U_c|} \leq -0.05$; open regions, $\frac{|u'_{sw}|}{|U_c|} \geq 0.05$.

flow at the local turbulent velocity using Taylor's hypothesis (i.e. let $x_1 = x_0 + \bar{u}_{turb}(t_1 - t_0)$. However, it is known that in general Taylor's hypothesis does not hold for large scales (Kim and Adrian, 1999).

In Couette flow the laminar and turbulent velocity profiles always overlap at the centerline, so the centerline velocity (U_c) is not affected by any assumptions of the $2D/3C$ model. The centerline also represents the wall normal location where the temporal fluctuations are at a minimum. For these reasons, it is the most natural location to study first. In the experiment of Kitoh and Umeki (2008) the authors compared their convected velocity to a spatial flow visualization and determined that at the centerline the large scales do in fact convect at U_c. Given their results we use the same relationship $x = x_0 - U_c t$ to transform our $2D/3C$ time series data into spatial data. In their work they define the Reynolds number $Re_c = \frac{U_c \delta}{\nu} = 4Re_w$ based on the channel half-height δ and the velocity at the centerline U_c, so the discussion in this section refers to both Re_w and Re_c.

Figure 3 shows the typical streak pattern on the central plane ($\frac{y}{\delta} = 1$) of Couette flow obtained using the $2D/3C$ model for Case 2, at $Re_w = 12800$, with $\sigma_{noise} = 0.004$. For visualization purposes and for direct comparison with the results of Kitoh and Umeki (2008) we similarly define a streak as a region where $|\frac{u'_{sw}}{U_c}| \geq 0.05$. Dark regions are low-speed streaks and open areas are high speed streaks, the light grey regions indicate a neutral region. It is clear that Cou-

ette flow generated using the $2D/3C$ model has significantly long streaks in the core region that are qualitatively similar to large scale features that have been identified through full three dimensional simulations and experiments.

Previous results (Komminaho et al., 1996; Tillmark and Alfredsson, 1998; Tsukahara et al., 2006;) have estimated streaks with streamwise wavelength of $\approx 40\delta - 64\delta$ with spanwise spacing of $\approx 2\delta - 5\delta$. Figure 3 shows that the spanwise lengthscale of our data is similar to these results. The streamwise extent of the structures produced by our model is much longer than reported in other works. This is not surprising as one would expect the results from the $2D/3C$ model to be more coherent than experimental data since we are only modeling large scale behavior.

In the vortex generator case of Kitoh and Umeki (2008) the authors also found that the streamwise lengthscale of the structures was approximately $51\delta - 60\delta$. However, when they attempted to isolate the large scale structures using a wavelet analysis they found that the $\sim 60\delta$ streaks form weakly wavy patterns that come together to form larger spatial structures with an average spacing of $300\delta - 400\delta$. Figure 15 from this work is shown here as Figure 4. Here, it is clear that these wavy patterns visually appear as one long streak with an extent $> 250\delta$.

It is possible that because our model essentially averages out the small scale affects it is not possible to distinguish between the long wavy structures reported in Kitoh and Umeki (2008) and the smaller lengthscale structures that they are

comprised of. The coarse grid in space and time that were used in the results reported herein may also be a reason for our inability to pick out the individual streaks. In the first DNS of Couette flow Lee and Kim (1991) also found structures extending about 1000δ. Those conformations were stationary in both space and time and it has been suggested in the literature that insufficient resolution was the cause of the extra coherence in their results. It is also possible that since we are only modeling the mean (large scale) behaviour, convecting at the local turbulent mean velocity may be introducing effects from the temporal fluctuations. Further work is needed to determine the true cause of the increased coherence in our results.

CONCLUSIONS

Streamwise constant structures have long been shown to have a significant role in both transition to turbulence and in fully developed turbulent flows. Our results show that a streamwise constant projection of the NS equations (the $2D/3C$ model) captures the mean velocity profile of fully developed turbulent plane Couette flow at these low Reynolds numbers. It should be noted that the disagreement in the mean velocity profile between the model and DNS data is within the error of our simulation method based on the coarse grid that was selected for this study. Future work involves characterizing the effect of grid size on the simulation results.

The preliminary results presented here also illustrate the ability of the model to capture streamwise elongated streaky structures in the core. A finer spanwise grid and smaller time steps may improve the resolution of our results. Further, study of the structures in the core using the $2D/3C$ model may give us new insight into the nature of these structures because in full simulations the large scale structures are disturbed by small scale turbulent motions. In the $2D/3C$ model these small scale motions are not present.

These results are especially promising because the analytical tractability of this model makes it well suited to studying behavior at larger Reynolds numbers. The ability to look at larger Reynolds numbers opens up many possibilities for future study.

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