Three-dimensional fully unsteady computational fluid dynamic simulations of five Olympic-level swimmers performing the underwater dolphin kick are used to estimate the swimmer’s propulsive efficiencies. These estimates are compared with those of a cetacean performing the dolphin kick. The geometries of the swimmers and the cetacean are based on laser and CT scans, respectively, and the stroke kinematics are based on underwater video footage. The simulations indicate that the propulsive efficiency for human swimmers varies over a relatively wide range from about 11% to 29%. The efficiency of the cetacean is found to be about 56%, which is significantly higher than the human swimmers. The computed efficiency is found not to correlate with either the slender body theory or with the Strouhal number.

Keywords: propulsive efficiency, propelling efficiency, dolphin kick, CFD

1 Introduction

As the name suggests, the dolphin kick resembles the type of motion adopted by dolphins and other cetaceans, where the key feature is an undulatory body wave motion that is initiated in the upper part of the body and which increases in amplitude as it propagates down toward the lower extremities. In the fully submerged dolphin kick, which is allowed during starts and turns in competitive swimming, the arms are outstretched and locked together in a streamlined position ahead of the head and the body usually remains symmetric about the sagittal plane during the entire stroke. The swimmer may be oriented with their chest facing up, down, or sideways (also called dorsal, ventral, and lateral positions) depending on the particular stroke and individual preference.

Propulsive efficiency of human swimming has been a topic of great interest in the swimming biomechanics community. Propulsive efficiency \( \eta \) is defined as

\[
\eta = \frac{W_{\text{useful}}}{W_{\text{total}}}
\]

where \( W_{\text{total}} \) and \( W_{\text{useful}} \) are the total and useful work, respectively, done by the swimmer over one stroke. Direct measurements of propulsive efficiency are virtually impossible to obtain since the work done by the swimmer cannot be measured in experiments. Thus, studies generally use experimental methods combined with hydrodynamic models to estimate the total efficiency, where the total efficiency is the product of the propulsive efficiency and the aerobic efficiency. The aerobic efficiency is the ratio of total mechanical work to energy expenditure (as determined by oxygen consumption above resting [1,2]. Kemper et al. [1] reported total efficiency results for skilled and unskilled swimming subjects performing the front crawl to be 4.76–4.87% and 2.96–3.33%, respectively. Later, Kemper et al. [2] reported values of 5.39% and 3.88% for skilled and unskilled swimmers, respectively. Adrian et al. [3] noted that the efficiency of the leg kick in the front crawl stroke ranged from 0.05% to 1.23%, that of the arm stroke ranged from 0.56% to 6.92%, and that of the whole stroke ranged from 1.71% to 3.99%. Toussaint et al. [4] used the measurement of active drag (MAD) system and oxygen consumption in determining the efficiency of a quasifreestyle stroke. In this system the swimmer uses underwater push-off pads to generate forward thrust. The push-off pads measure forces generated by the swimmer’s arms. Since the swimmer is using push-off pads instead of the water for propulsion, the MAD system does not recreate the true freestyle stroke. They reported a mean total efficiency of 5.4% and a mean propulsive efficiency of 58%.

Obtaining even rough estimates of the efficiency for the fully submerged dolphin kick is especially challenging since systems like MAD cannot be used for this purpose. Nicolás et al. [5] used the idealized slender body theory [6] to estimate the propulsive (or Froude) efficiency for 12 international-level monofin swimmers performing the submerged dolphin kick. According to this highly idealized theory, the slender body theory efficiency \( \eta_s \) is computed as

\[
\eta_s = \frac{(c + V)}{2c}
\]

where \( c \) is the wave speed of the body wave and \( V \) is the average swimming speed. Based on this theory, Nicolás et al. [5] reported a mean efficiency of 79% for the monofin swimmers. It should be noted that this theory neglects the drag on the body and is therefore expected to overestimate the efficiency. Nevertheless, this estimate was surprising given that the efficiencies reported for dolphins (specifically, bottlenose dolphins (Tursiops truncatus)) are about 86% [7].

Given that the dolphin kick is an important component of competitive swimming, it is of great interest to obtain better estimates of propulsive efficiency for this stroke. Motivated by this, we have used computational fluid dynamics (CFD) to obtain direct estimates of this quantity for a number of Olympic-level swimmers and compared these direct estimates to those for a cetacean and also to the slender body theory.

2 Method

A finite difference CFD solver is used to solve the viscous incompressible Navier–Stokes equations on a fixed nonuniform Cartesian grid using an immersed boundary method [8]. This solver has been extensively documented for a variety of flows associated with aquatic propulsion [9–11]. The simulations are fully unsteady and no quasisteady assumptions are made. All the current simulations were carried out on a dense Cartesian grid with about \( 4.2 \times 10^6 \) mesh points with the highest resolution concentrated around the body and its wake. The current solver is limited due to the computational costs to Reynolds numbers of about \( 10^6 \), which is two orders of magnitude lower than the actual Reynolds number.
The swimmers are fully submerged and there is no air-water interface in the simulations. The simulations are performed in a reference frame moving at the average speed of the swimmer. The no-slip condition is enforced at the body surface. Simulations are performed on a single processor of a 2.0 GHz AMD Opteron and take on the order of 10 days of computational time per stroke cycle. Multiple contiguous strokes are simulated and the analysis is based on the later strokes. Further details of the numerical method and modeling procedure can be found in Ref. [12]. As described in Ref. [13], test simulations were also carried out on a grid with $6 \times 10^6$ mesh points in order to assess the grid dependency of the results. Key parameters such as mean horizontal and transverse forces were found to vary less than 5% and thus, the grid with $4.2 \times 10^6$ mesh points was considered acceptable for the current study.

The surface shape of the swimmer’s body is based on accurate laser scans of two actual Olympic-level athletes (one female and one male) whereas the body of the cetacean is based on a computerized tomography (CT) scan of a harbor porpoise. For computational purposes, this surface is converted into an unstructured triangular surface mesh with 10,000–30,000 triangular elements. This surface is scaled to fit the different swimmers in the video footage. In particular, the body of the harbor porpoise was scaled up to the size of a killer whale, since good quality video footage of a swimming killer whale was available. Prerecorded video footage of Olympic-level athletes performing the dolphin kick is used in combination with animation software to reconstruct the body motion for inclusion into the simulations. Table 1 lists the salient kinematic parameters for the various swimmers considered in the current study. These were extracted through analysis of the video footage of the swimmers. We also estimate the Strouhal number for the swimmers since this nondimensional parameter has been closely associated with swimming efficiency [14, 15]. Finally, we used the video footage to track the progression of the wave on the body of the swimmers and also to estimate the forward speed of the swimmers. This allowed us to obtain $c$ and $V$ for the estimation of $\eta_k$, as shown in Eq. (2). A detailed discussion of this kinematic analysis can be found in Ref. [16]. Figure 1 shows the typical vortex structure seen in the dolphin kick simulations as expressed by one isosurface of enstrophy for the cetacean and for the Female-1 case.

The numerical solution of the Navier–Stokes equations allows us to compute the flow and pressure through the entire time and space domain. Specifically, we can compute the pressure and the shear forces acting on each segment of the body. With this knowledge, calculation of the propulsive efficiency is straightforward. Integration of the pressure and the shear over an individual triangular element (say, triangle number $n$) of the body surface gives us the force $F_n(t)$ that this surface triangle is exerting on the flow at time $t$. Furthermore, let $U_n(t)$ be the velocity of this surface triangle with respect to the laboratory (fixed) reference frame at this time instant (see Fig. 2). Then, the total work done by the swimmer over one stroke is computed as

$$W_{\text{total}} = \int_0^\tau \sum_{n=1}^N F_n(t) \cdot U_n(t) dt$$  \hspace{1cm} (3)

where $\tau$ is the time period of the stroke and $N$ is the total number of surface triangles. The useful work done during one stroke can be computed as

$$W_{\text{useful}} = \int_0^\tau \sum_{n=1}^N \left( \frac{F_{nX} - |F_{nX}|}{2} \right) U_{nX} dt$$  \hspace{1cm} (4)

where subscript $X$ denotes the vector component in the direction of the swimmer’s travel and the expression in the parentheses in Eq. (3) extracts the force only for those surface triangles, which exert a force on the flow in the downstream direction. Note that the summation in the above expressions represents the instantaneous power $P$.

### 3 Results and Discussion

Table 2 lists the computed values of the mean useful power, mean total power, useful work, total work, and propulsive efficiency. Also included is the Froude efficiency estimated based on

<table>
<thead>
<tr>
<th>Female 1</th>
<th>Male 1</th>
<th>Male 2</th>
<th>Female 1</th>
<th>Female 2</th>
<th>Female 3</th>
<th>Cetacean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_m$ (m/s)</td>
<td>0.95</td>
<td>0.95</td>
<td>0.97</td>
<td>0.97</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>$f$ (Hz)</td>
<td>1.81</td>
<td>2.63</td>
<td>2.22</td>
<td>2.22</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>$A$ (m)</td>
<td>0.64</td>
<td>0.48</td>
<td>0.60</td>
<td>0.74</td>
<td>0.58</td>
<td>0.74</td>
</tr>
<tr>
<td>$c$ (m/s)</td>
<td>3.88</td>
<td>4.79</td>
<td>4.46</td>
<td>2.89</td>
<td>5.07</td>
<td>6.32</td>
</tr>
<tr>
<td>$l$ (m)</td>
<td>2.14</td>
<td>1.82</td>
<td>2.01</td>
<td>1.50</td>
<td>2.13</td>
<td>4.17</td>
</tr>
<tr>
<td>$S_{St}$</td>
<td>1.22</td>
<td>1.37</td>
<td>1.05</td>
<td>1.20</td>
<td>1.05</td>
<td>0.18</td>
</tr>
</tbody>
</table>

![Fig. 1 Typical vortex structures seen in (a) the Cetacean stroke and (b) the human dolphin kick](image-url)
the slender body theory. We find that the mean total power predicted is well within observable range for humans. For instance, Sargeant et al. [17] determined the peak power output produced by one leg acting alone in an alternating cyclical movement to be about 1387 W.

Mean useful work varies considerably from about 23 J to 60 J among the human swimmers. Computed propulsive efficiency values also span a broad range from about 11% to 29% for the humans. These are significantly higher than the total efficiency values estimated for the front crawl. Unfortunately aerobic efficiency values are not known for the current subjects and so no total efficiency values are presented. However, high dolphin kick efficiency values might point to the advantage of avoiding wave drag by swimming underwater [18,19]. The Froude efficiency values based on the slender body theory are significantly higher and spread over a narrower range from 60% to 70% for the humans. These numbers are inline with those reported by Nicolas et al. [5] for monofin swimmers. It should be noted that apart from the mismatch in magnitudes, there is also no clear correlation between the CFD based efficiency and that predicted using the slender body theory. Furthermore, there is also no correlation between the Strouhal number and propulsive efficiency. This is a clear manifestation of the fact that propulsive efficiency in the human dolphin kick does not simply depend on gross kinematic parameters but is the result of the overall swimming style, which encompasses many different aspects of the body motion. The current simulations also point to the limitation of the slender body theory as applied to human swimming.

As expected, the propulsive efficiency (η) of the cetacean, which is about 56% in this study, is significantly higher than the human swimmers. The higher efficiency of dolphins is due to two factors: the first is the enhanced thrust generation by the large span of the propulsive flukes and the second is the generation of a very smooth wave along the body (due to multiple joints in the body and tail, i.e., the vertebrae), which along with the streamlined shape leads to a very low drag coefficient during swimming [7,20]. A lower drag during swimming directly results in a decrease in the total used power. Due to a lower drag, the slender body theory should also provide a better approximation for this case than it does for the human swimmers. This is confirmed by the fact that the slender body theory predicts an efficiency (η) of 75% for the cetacean, which is relatively close to the value predicted by the CFD.

4 Conclusions

Computational fluid dynamics has enabled direct calculation of work, power, and propulsive efficiency for the fully submerged dolphin kick. Mean power expended over the course of one stroke for the human subjects was 424 W. The total work done over the course of one stroke was 199 J, and the mean propulsive efficiency was about 21%. Such a high value of propulsive efficiency is most likely due to the absence of wave drag and re-affirms the hydrodynamic advantage of swimming underwater. The cetacean showed a higher efficiency of 56%. Computed propulsive efficiencies exhibit no clear correlation with efficiencies predicted from the slender body theory or with the Strouhal number. This indicates that propulsive efficiency of the dolphin kick in humans depends not on any simple kinematic parameter(s) but is connected with overall swimming style, which encompasses many different aspects of the body motion.

References


Table 2 Mean useful and total power, useful and total work, and propulsive efficiencies for the submerged dolphin kick

<table>
<thead>
<tr>
<th></th>
<th>$\bar{P}_{\text{useful}}$ (W)</th>
<th>$\bar{P}_{\text{total}}$ (W)</th>
<th>$W_{\text{useful}}$ (J)</th>
<th>$W_{\text{total}}$ (J)</th>
<th>$\eta$ (%)</th>
<th>$\eta_{\text{hydro}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female 1</td>
<td>42.5</td>
<td>290.4</td>
<td>23.4</td>
<td>159.7</td>
<td>14.7</td>
<td>62.2</td>
</tr>
<tr>
<td>Female 2</td>
<td>85.1</td>
<td>289.8</td>
<td>32.1</td>
<td>109.3</td>
<td>29.4</td>
<td>59.9</td>
</tr>
<tr>
<td>Female 3</td>
<td>77.6</td>
<td>416.3</td>
<td>34.9</td>
<td>187.4</td>
<td>18.6</td>
<td>60.9</td>
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<tr>
<td>Male 1</td>
<td>71.8</td>
<td>639.0</td>
<td>37.3</td>
<td>332.3</td>
<td>11.2</td>
<td>70.4</td>
</tr>
<tr>
<td>Male 2</td>
<td>141.7</td>
<td>482.7</td>
<td>59.9</td>
<td>204.2</td>
<td>29.3</td>
<td>62.9</td>
</tr>
<tr>
<td>Cetacean</td>
<td>191.3</td>
<td>3426.3</td>
<td>126.2</td>
<td>2261</td>
<td>55.8</td>
<td>75.3</td>
</tr>
</tbody>
</table>

Fig. 2 Total and streamwise component of force exerted by one representative triangular surface element. The velocity of the triangle in the laboratory reference frame is also shown and the swimming direction and speed $V$ is indicated by the gray, horizontal arrow.


