Generation of Streamwise Vortical Structures in Bluff Body Wakes

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Direct numerical simulation is used to study the periodic generation and evolution of streamwise vortical structures in the near wake of a circular cylinder. It is observed that streamwise structures are formed due to stretching of core vorticity which escapes out of the core and due to stretching of small scale streamwise vorticity already present outside the core. Distinct hairpin structures are observed which play a central role in transferring vorticity out of the core. Furthermore, the hairpin structures are associated with a spanwise subharmonic mode and are thus manifestations of a period-doubling mechanism.

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The generation of streamwise oriented vortical structures, also referred to as *ribs*, marks the onset of three dimensionality in free shear layers and wakes. A number of studies [1–12] have focused attention on understanding the underlying mechanisms of this three-dimensional (3D) transition. However, no general consensus has been reached, and some studies attribute the generation of ribs to a *braid instability* [2–7] , whereas others attribute it to a *core instability* mechanism [1,8–10].

Although precise definitions of the braid and core instability mechanisms are lacking in the existing literature, in general terms braid instability refers to the mechanisms whereby small scale perturbation in the braid region are stretched due to the local strain field and lead to the formation of streamwise vortical structures. During this process, the cores of the spanwise vortices remain primarily two dimensional (2D), and subsequent spanwise deformation of the cores is due to the interaction with the mature ribs. In contrast, core instability refers to a mechanism whereby the cores of the spanwise vortices undergo distortion in the spanwise direction. Subsequently, the distorted core interacts with the braid region and leads to the generation of streamwise structures.

The above mechanisms are easier to distinguish in temporal evolution of shear flows, where the 3D perturbations grow in time and lead to the generation of streamwise structures. In numerical simulations, this corresponds to studying the temporal evolution of 3D disturbances in streamwise periodic domains. In the case of experiments, such studies would correspond to analyzing the self-similar regime far downstream of the splitter plate or the cylinder. Most such temporal studies [2–5] attribute a braid instability mechanism to the generation of streamwise structures. One possible reason might be that background or initial 3D perturbations present in the braid region get amplified faster than in the core and lead to the generation of streamwise structures well before the 3D distortion of the spanwise cores.

On the other hand, in spatially developing flows where the streamwise structures are periodically generated just downstream of the splitter plate or the cylinder, experimental studies show that the distortion of the core plays an integral role in the generation of the streamwise structure [8–10]. The basic difference between temporally and spatially developing flows is that in temporally developing flows the growth of all the streamwise vortical structures occurs simultaneously over time, whereas in spatially developing flows new streamwise structures are generated in the presence of other well-developed streamwise structures and distorted cores which were part of the previous cycle. Thus the presence of a region of strong absolute instability and the resulting process of autogeneration of streamwise vortical structures in the near wake of bluff bodies are significantly different.

In line with the experimental visualizations of Williamson [10], we observe that the rollers deform significantly as they roll up. Furthermore, it is found that this deformation is caused by the action of the streamwise structures that were generated in the previous shedding cycle. It is also clear that the stretching of streamwise vorticity in the wake cavity due to the action of the rollers is an integral part of the generation process. Thus one essential issue that needs to be investigated is the origin of the streamwise vorticity in the wake cavity. Does it result from a rearrangement and collapse into the compact core of the already existing small-scale streamwise vorticity arising from 3D perturbations, or does vorticity from the core escape into the braid region and get stretched to form the streamwise ribs? Although experimental flow visualizations of Nygaard and Glezer [8,9] and Williamson [10] clearly show the dye escaping from the core into the braid forming streamwise riblike structures, the possibility of core vorticity entering the braid is not completely established.

In this Letter, we address the above question by following the evolution of streamwise structures in the near wake of a circular cylinder at a Reynolds number based on the diameter of 525. The simulation is based on a highly accurate Fourier-Chebyshev collocation spectral method, details of which can be found in Ref. [12]. The spanwise

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length of the computational domain is taken to be equal to the cylinder diameter, and periodic boundary conditions are imposed along the span. At this Reynolds number, the wake has an organized structure in the spanwise direction with a spanwise wavelength of the order of the cylinder radius [13,14] and in the present simulation two pairs of counter-rotating streamwise ribs are obtained. Average drag, base pressure coefficient, and Strouhal number obtained from the present simulation are 1.24, −0.92, and 0.22, respectively, and these are in good agreement with established experimental values (see Ref. [15]).

Proper identification and extraction of vortical structures are important in understanding their origin and dynamics. We observe that for the 3D flow field encountered in the near wake, identification through the magnitude of the imaginary part of the complex eigenvalue (λ_i) of the velocity gradient tensor is an effective means of extracting vortical structures [16–18]. This method is frame invariant and identifies a "vortical structure" as the region of large vorticity, where rotation dominates over strain, thus correctly eliminating shear layers from consideration. Thus vortical structures identified using this method approximately correspond to circular streamlines in planes perpendicular to the axis of the structures.

In Fig. 1 a sequence of four isosurface plots of λ_i is presented which shows the evolution of the vortical structures in the near wake over half of a shedding cycle. The maximum magnitude of λ_i is found to be in the range 4.0 to 5.0, and in all the plots the isosurface corresponds to a value of $|\lambda_i| = 1.1$. Top, bottom, and side views have been shown, and the flow is from left to right with the base of the cylinder shown on the left side of each plot. In the side views, superposed on the isosurface plot is also shown one contour level of spanwise vorticity magnitude at one local spanwise cross section so as to provide the reader with a clear visual reference. In all these views, only a section of the computational domain in the near wake is shown.

Figure 1(a) shows the bottom shear layer in an early stage of its rollup into a counterclockwise roller (CCR1) and the attached clockwise roller (CR1) in the process of separating from the shear layer. A separated counterclockwise roller (CCR2) can also be observed in the figure at a downstream location and is connected to CCR1 by four ribs (R1). The direction of the rotation of these ribs is shown by arrows in the top view. We can also observe remnants of ribs (R2) that connect CCR2 with the downstream clockwise roller (CR2) which is not shown in the figure.

A number of observations can be made from this figure. (1) CR1 shows large spanwise distortions long before it separates from the shear layer. Of particular note is the crescent shaped feature which is seen embedded in CR1 in the top view. (2) The top view also shows a distinct corrugated structure in the newly forming CCR1. These corrugations are induced by the action of the ribs, which alternately stretch and compress this spanwise roller, thereby

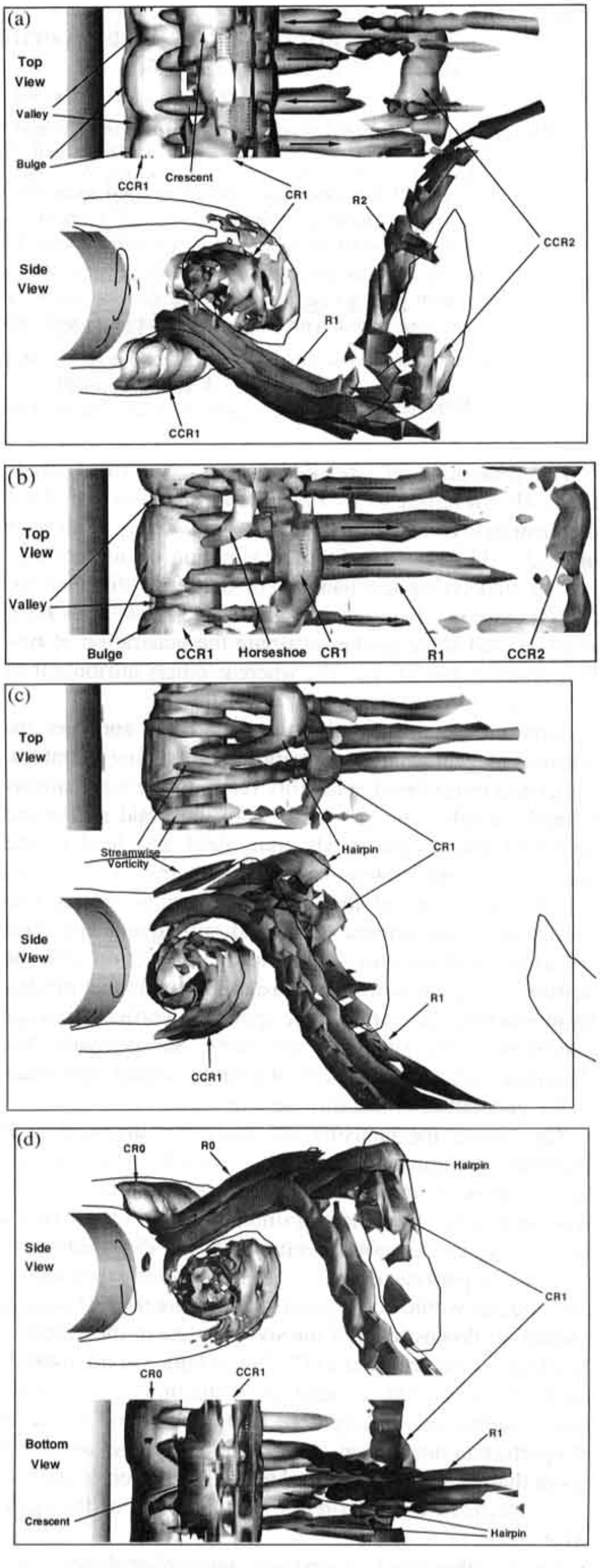

FIG. 1. Sequence of four plots showing vortical structures in the near wake visualized by $|\lambda_i| = 1.1$ over a time interval corresponding to about half of the shedding cycle.

increasing the strength in some regions forming"bulges" and reducing the strength in between forming "valleys". A similar interaction has been observed and explained by Rogers and Moser [11] in the context of a temporally growing shear layer.

Figure 1(b) is a top view corresponding to a later time when both CR1 and CCR1 have grown in size and CR1 has moved further downstream, and we observe the following. (1) CR1 has split into two separate structures, and the main part of the roller has moved downstream and reduced in size. (2) The crescent shaped feature has broken off from CR1 and has developed into a "horseshoe" structure. (3) The bulges and valleys in CCR1 have become more pronounced due to the sustained action of the ribs, R1, and the two bulges are not of equal size, with the bulge in the center being wider and stronger than the one at the outer edges.

Figure 1(c) shows the top and side views at a still later time when CCR1 has grown to its full strength and CR1 is in the final stage of pinch-off from the separated shear layer. We observe the following. (1) the horseshoe structure, which is situated in the "neck" region between the clockwise roller and the associated separated shear layer, has elongated into a "hairpin" structure similar to those observed in boundary layers and has clearly separated from CR1. This growth is due to the intense stretching associated with the neck region of CR1. It should be pointed out that subsequently the neck region will become depleted of spanwise vorticity and form the braid between CR1 and the new clockwise roller. In contrast to similar structures observed in plane and bluff body wakes [5,10], the head of the hairpin is never observed to wrap around the downstream (CR1) roller. (2) CCR1 continues to deform and rotate in the counterclockwise direction, and the bulges that were clearly observable in Fig. 1(b) are now hidden from view. (3) We observe small-scale streamwise structures in the wake cavity. These structures are not connected to any other structures and appear to form from the stretching of small-scale streamwise vorticity in this region.

Figure 1(d) is the last of this sequence, and the side and bottom views are shown. We observe the following. (1) CR1 has been shed, and the top shear layer is rolling into a new counterclockwise vortex (CR0) which is already distorted in the spanwise direction. (2) The pairs of counter-rotating streamwise ribs (R0) have grown in strength and size. One of the two pairs (marked in the bottom view) is formed due to the stretching of the legs of the hairpin vortex, whereas the other pair is formed due to stretching of small-scale streamwise vorticity. (3) The bottom view clearly shows the emergence of the familiar crescent shaped feature in CCR1. By following the evolution of CCR1, it is observed that the crescent shaped feature is formed directly out of the big bulge in the center of CCR1. This incipient crescent shaped feature later develops into a horseshoe vortex which gets stretched into a hairpin vortex. A new pair of counter-rotating ribs is subsequently formed due to the stretching of the legs of this hairpin, and simultaneously another pair of ribs are also generated out of the small-scale disturbance in the neck region. Thus the above described scenario is not an isolated set of events but is repeated over every shedding cycle.

Thus hairpin structures are formed out of each roller; however, it can be observed from Fig. 1 that the bulges and the associated hairpin structures they generate are formed out of clockwise and counterclockwise rotating cores and are also displaced by half a wavelength along the span. From the spanwise periodicity, it can be seen that the hairpin structure appears at the same spanwise location only on alternate shedding cycles. This spanwise subharmonic behavior implies a period-doubling scenario to one suggested in previous studies [19,20]. Interestingly, the present simulations show that the period doubling is accomplished by the appearance of a distinct vortical structure, i.e., the hairpin vortex.

Based on experimental visualizations, Williamson [10] has conjectured that streamwise vortices are mainly formed out of core vorticity that has been pulled into the braid region. The evolution of the hairpin vortices in the present simulations clearly demonstrates one possible mechanism for vorticity entrainment from the core into the neck region. New rollers are deformed in the near wake due to the sustained action of the ribs of the previous generation, and pieces of the deformed core break off and enter the neck region, where they get stretched to form streamwise structures. In addition, it is observed that ribs are also formed due to the stretching of small-scale streamwise vorticity that exists in the wake cavity. The origin of this small-scale streamwise vorticity is still not clear; however, one possible mechanism could be induction due to the legs of the hairpin in the presence of shear in a way postulated by Lasheras, Cho, and Maxworthy [3].

With regard to the question of braid vs core instability, it should be emphasized that the first appearance of the streamwise structures is sufficiently close to the body in the near wake that the rollup of the shear layer is not complete, and a well defined braid region cannot be identified. Moreover, we observe that the stretching responsible for the inception and initial enhancement of the streamwise vortices is [illegible] due to the straining action of the attached roller and the associated separated shear layer. Only sufficiently far downstream are the rollers arranged in a staggered fashion, and the stretching is associated with the straining action of the two adjacent rollers [5]. This downstream topology is more closely related to the corresponding temporal problem studied previously [2–5]. Thus the current simulations complement the temporal studies by explaining the mechanism of autogeneration of streamwise structures in the near wake.

These computations have been performed on the C-90 at the Pittsburgh Supercomputing Center.

[1] R.T. Pierrehumbert and S.E. Widnall, J. Fluid Mech. **114**, 59 (1982).
[2] G.M. Corcos and S.J. Lin, J. Fluid Mech. **139**, 67 (1984).
[3] J.C. Lasheras, J.S. Cho, and T. Maxworthy, J. Fluid Mech. **172**, 231 (1986).
[4] J.C. Lasheras and H. Choi, J. Fluid Mech **180**, 53 (1988).
[5] E. Meiburg and J.C. Lasheras, J. Fluid Mech. **190**, 1 (1988).
[6] L.P. Bernal, Ph.D. thesis California Institute of Technology, 1981.
[7] L.P. Bernal and A. Roshko, J. Fluid Mech. **170**, 499 (1986).
[8] K.J. Nygaard and A. Glezer, Phys. Fluids A **2**, 461 (1990).
[9] K.J. Nygaard and A. Glezer, J. Fluid Mech. **231**, 257 (1991).
[10] C.H.K. Williamson, *Turbulent shear Flows 7* (Springer-Verlag, Berlin, 1991), p. 173.
[11] M.M. Rogers and R. Moser, J. Fluid Mech. **243**, 183 (1992).
[12] R. Mittal and S. Balachandar, UIUC TAM Report No. 751.
[13] J. Wu *et al.*, J. Fluids Struct. **8**, 621 (1994).
[14] H. Mansy, P. Yang, and D.R. Williams, J. Fluid Mech. **270**, 277 (1994).
[15] R. Mittal and S. Balachandar (to be published).
[16] R. Mittal and S. Balachandar, Report No. AIAA 95-0867, 1995 (unpublished).
[17] U. Dallmann *et al.*, Report No. AGARD CP 494, 1991 (unpublished).
[18] J. Zhong, T.S. Huang, and R.J. Adrian, in Proceedings of IEEE CVPR Conference, Seattle, WA, 1994 (to be published).
[19] A. Tomboulides, G. Triantafyllou, and G.E. Karniadakis, Phys. Fluids A **4**, 1329 (1992).
[20] D. Rockwell, F. Nuzzi, and C. Magness, Phys. Fluids A **3**, 1477 (1991).