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A combined application of the integral wall model and the rough wall rescaling-recycling method

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In this study, a recently proposed integral wall model[1] and concurrent inflow generation technique[2] are applied in Large-Eddy-Simulation (iWMLES) of developing turbulent boundary layer flow over cuboidal roughness. We examine the performance of this integral wall model at various Reynolds numbers. The integral wall model is based on the von-Karman-Pohlhausen integral method. With several parameters in the proposed functional form of the velocity profile determined from the local flow conditions, the wall model predicts velocity profiles that satisfy the vertically integrated momentum equation. Only an algebraic system must be solved in the wall model which thus preserves the essential economy of equilibrium type models. The rough wall inflow generation technique is proposed based on a new definition of a length scale that is appropriate for the roughness dominated inner layer. It extends the rescaling-recycling method[3] to rough surfaces. The integral wall model and the rough wall rescaling-recycling method are applied in Large Eddy Simulations of turbulent boundary layers over surface with distributed cuboidal roughness. The effect of Reynolds number is studied. A good agreement is found between the roughness function (velocity shift) measured in iWMLES and the Colebrook formula[4] and previous experimental measurements.[5–8]

Nomenclature

a_L	roughness area density
A	linear correction coefficient for log law
B	smooth wall log law constant
C	velocity shift in log law
C_d	drag coefficient
d	zero-plane displacement height
f	distributed body force
h	roughness height
k_s	equivalent sandgrain roughness height
Re	Reynolds number
t	time
$\langle u \rangle$	filtered velocity
u_ν	inner layer velocity scale
u_τ	friction velocity
U_{LES}	LES velocity
U_0	free stream velocity
x	streamwise direction
y	wall normal direction
z	spanwise direction
z_o	hydrodynamic roughness height
δ	boundary layer thickness

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δ_0	inlet boundary layer thickness
δ_ν	inner layer length scale
δ_i	inner layer height
Δ_y	meso layer height
ΔU^+	roughness function
κ	von Karman constant
λ_f	roughness solidity
ν	kinematic viscosity
ρ	density
τ_w	wall stress

I. Introduction

Large Eddy Simulations (LES) of developing turbulent boundary layers over rough surfaces have direct applications to the prediction of drag and flow structure in many practical applications[9,10] . In this study, we present the combined application of the recently developed integral wall model and the rough wall inflow generation technique in LES of flow over surfaces with distributed cubiform roughness. The integral wall model is briefly described in section II(see Ref. 1 for more details). In section III, the rescaling-recycling inflow generation technique for rough surfaces is summarized. The applications are presented in section V, followed by the conclusions in section VI.

II. The integral wall model for LES

In this section, we summarize the integral wall model.[1] The model connects the resolved velocity at a distance from the wall to the wall stress. To include flow physics in the near wall region while preserving the basic economy of equilibrium-type wall models, a variant of the classical integral method of von Karman and Pohlhausen (VKP) is developed. A velocity profile with various parameters is proposed as an alternative to numerical integration of the boundary layer equations in the near-wall zone. The profile contains a viscous or roughness sublayer, and a logarithmic layer with an additional linear term that can account for inertial and pressure gradient effects. The profile reads:

$$\begin{aligned} \langle u \rangle &= u_\nu \frac{y}{\delta_\nu}, & 0 \le y \le \delta_i; \\ \langle u \rangle &= u_\tau \left[C + \frac{1}{\kappa} \frac{y}{\Delta_y} \right] + u_\tau A \frac{y}{\Delta_y}, & \delta_i \le y \le \Delta_y; \end{aligned} \tag{1}$$

where $\langle u \rangle$ is the velocity filtered in wall-parallel planes at LES scales, y is the wall normal direction, δ_i is the inner layer height and Δ_y is the distance from the wall where LES velocity is available. u_ν and δ_ν are the 'inner layer' velocity and length scales, respectively. u_τ is the velocity scale appropriate for the 'meso-layer'. A is a coefficient of the linear correction term and C is a constant from the log law. We restrict the discussion to 1-D for simplicity, i.e. we do not include the spanwise velocity component, $\langle w \rangle$, in the discussion.

Then 6 unknowns parameters, u_ν, δ_ν, δ_i, u_τ, A, and C need to be determined from local flow conditions. As in the VKP method, the profile shape function is substituted into the vertically integrated momentum equation, and with 5 other physical constraints and boundary conditions, the 6 unknown parameters can be solved from coupled algebraic equations. We briefly discuss those conditions here. First, we must match with the LES velocity, i.e. $\langle u \rangle (y = \Delta_y) = U_{\mathrm{LES}}$. Second, we impose continuity of the velocity profile at $y = \delta_i$, i.e. $\langle u \rangle \left(y = \delta_i^-\right) = \langle u \rangle \left(y = \delta_i^+\right)$. Third, we must specify the scale separating the two layers, δ_i. For fully rough surfaces with roughness elements protruding up to a height k, we choose $\delta_i = k$. Conversely, for smooth surface cases, δ_i represents standard separation between the viscous and the inertial layer, i.e. $\delta_i = 11\nu/u_\tau$, the intercept between the linear viscous profile $\langle u \rangle = y u_\tau^2/\nu$ and the standard log-law $\langle u \rangle = u_\tau/\kappa \ln(y u_\tau/\nu) + B$ with $B = 5$, $\kappa = 0.4$ and ν the kinematic viscosity. In the case of low local Reynolds numbers or, equivalently, in the case of wall-resolved LES, we may have $11\nu/u_\tau > \Delta_y$. In that case, the linear profile is assumed to extend all the way to Δ_y. Therefore, in order to include all cases, we define $\delta_i = \min\left[\max\left(k, 11\nu/u_\tau\right).\Delta_y\right]$. Fourth, the inner layer length scale is defined in terms of viscosity and near wall velocity scale: $\delta_\nu = \left(\nu + \nu_T\right)/u_\nu$. Fifth, we define the friction velocity u_τ associated with the total wall momentum flux as the sum of the viscous drag at the surface and the form drag implied by the distributed body force according to $\langle f_x \rangle = -C_d a_L \langle u \rangle^2$, where C_d and a_L are drag coefficient and roughness area density, respectively: $\tau_w = u_\tau^2 = u_\nu^2 + \int_0^k C_d a_L \langle u^2 \rangle dy$. u_τ is the velocity scale used in the meso-layer profile. We recall that $u_\nu = u_\tau$ for smooth walls. Finally, the vertically integrated momentum equation provides a condition that closes the coupled set of 6 equations for the 6 unknown: $dL_x/dt + M_x = \tau_{\Delta_y} - \tau_w$, where $L_x = \int_0^{\Delta_y} \langle u \rangle dy$, M_x contains the vertically integrated convective term and pressure gradient term, τ_{Δ_y}, τ_w are the momentum flux at $y = \Delta_y$ and at the wall, respectively. M_x, τ_{Δ_y} and τ_w can all be calculated from the velocity profile.

In practice, we use forward Euler for temporal discretization with time step dt for the unsteady term dL_x/dt. All information is assumed to be known at step n. The wall stress is calculated for step $n+1$ and fed back to LES. In the 'wall-resolving' case $\delta_i = \Delta_y$, we obtain $\tau_w = \nu U_{\mathrm{LES}}/\Delta_y$ by solving the conditions using the linear profile. In addition to the spatial filtering inherent in LES, we also apply a one-sided exponential relaxation temporal filtering with a time-scale $T_{\mathrm{wall}} = \Delta_y/\kappa u_\tau$ onto the variables and dynamical equations: $\langle u \rangle = \int_{-\infty}^{t} u \ \exp[-(t-t')/T_{\mathrm{wall}}]/T_{\mathrm{wall}} \ dt'$. This filtering operation is meant to represent the appropriate (long) time-scale associated with vertical turbulent and laminar diffusion in the near-wall region. These filtered velocities depend mainly on wall normal direction y, while a 'slow' spatial dependence on x, z at LES resolution, and dependence of time over time-scales on the order of T_{wall} also exists. More details on the need and rationale for the time-filtering can be found in Ref. 1.

III. The rough wall rescaling-recycling inflow generation technique

The rescaling-recycling method,[3] which was originally introduced for flow over smooth surfaces, is extended to rough wall boundary layers. Different from the case of smooth surfaces, where the inner layer length scale is ν/u_τ, the inner length scale for rough walls must be defined by the surface roughness. For the case of rough walls, we propose an inner length scale based on the normal dispersive stress in streamwise direction, i.e. the vertical distance by which the normal dispersive stress drops to 10% of its local maximum. Mean velocity and fluctuations at a downstream plane are then rescaled according to this newly proposed inner length scale and the usual outer length scale, the boundary layer thickness, δ. The rescaled velocity is then combined with a blending function for the inflow generation.

IV. Simulation Details

We use the in-house code Vicar3D to solve the incompressible filtered Navier-Stokes equations.[11] A second-order central finite difference scheme is used for spatial derivatives, and the projection method is employed for time discretization. The sub-grid stress is modeled with the Dynamic Vreman model.[12] The roughness is aligned cubes of height $h = 0.25\delta_0$ with solidity $\lambda_f = 0.06$ (figure 1). Here, δ_0 is the boundary layer thickness at the inlet The roughness elements are resolved with the sharp immersed boundary method.[13] The computational domain of size $16\delta_0 \times 4\delta_0 \times 4\delta_0$ in the x, y, z directions, respectively. The number of grid points is $256 \times 64 \times 64$. Periodic boundary conditions are used in the spanwise (z) direction. The top boundary condition is a zero gradient condition. The inflow velocity is prescribed via the extended rescaling-recycling method described in section III. A standard zero-diffusion condition is employed at the outlet. The code has been extensively validated.[11] A validation case that is particularly relevant is provided in Ref. 2. The free stream velocity U_0 is used as a velocity scale and the boundary layer thickness δ_0 at the inlet is used as a length scale for normalization. LES of developing turbulent boundary layers of Reynolds number $Re = U_0\delta_0/\nu = 10^3, 10^4, 10^5, 10^6$ is conducted, in order to explore the model's sensitivity to viscosity over a wide range.

V. Results

The temporally averaged velocity profile is further averaged in $x \in [7\delta_0, 9\delta_0]$ (two repetitive patterns) for the mean profile. Figure 2 shows the mean velocity profile. The skin friction is directly obtained within the simulations. The friction velocity $u_\tau = \sqrt{\tau_w/\rho}$ can then be directly calculated. A log law

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \log\left(\frac{y-d}{k_s}\right) + 8.5 \tag{2}$$

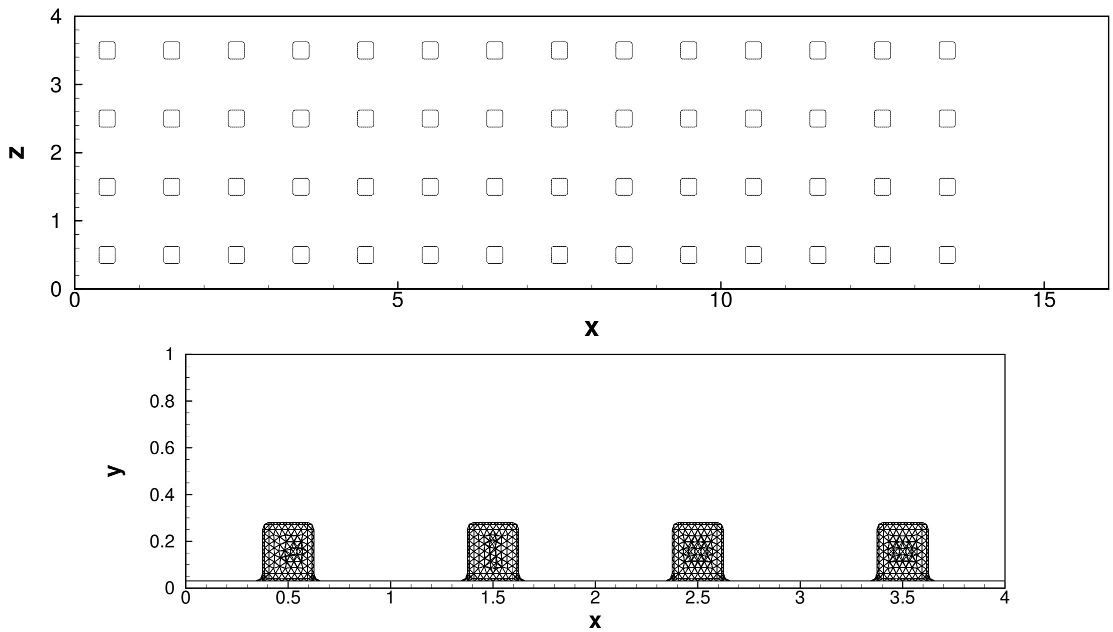

Figure 1. Top and (part of) side view of the surface roughness.

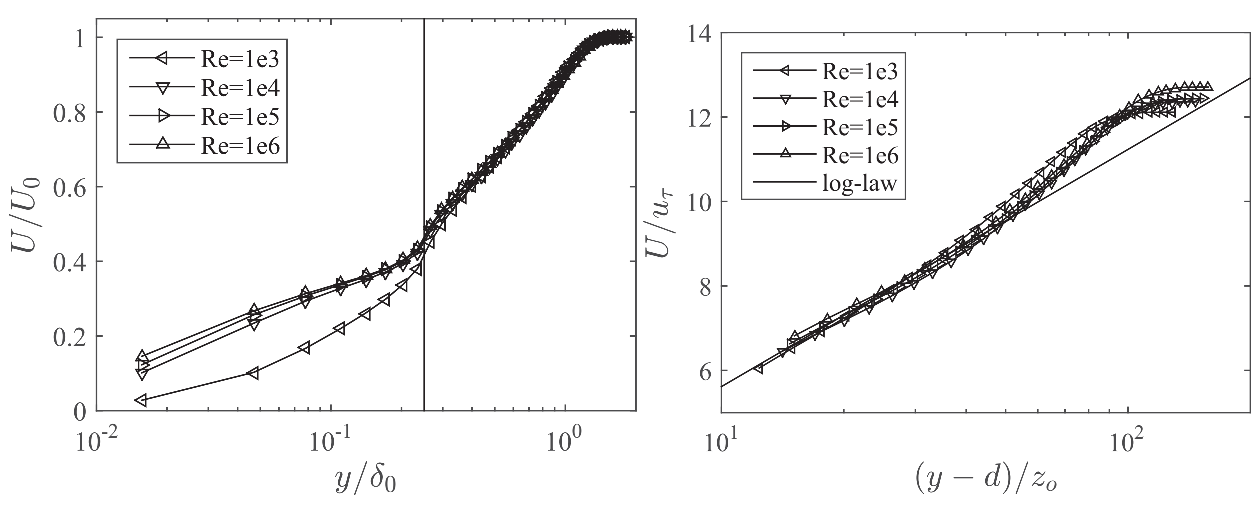

Figure 2. Mean velocity profile for all the cases. The height of the cubic roughness is marked with a thin solid line in the left figure. Left: outer units, right: using the fitted values of z_0 and d.

can be fitted to the velocity profile, where k_s is the equivalent sand grain roughness height, d is the zero-plane displacement height. Via fitting a log law at $Re = 10^6$ with u_τ known,[14] we have $k_s/h = 0.00129$, $d/h = 0.55$ through fitting. The hydrodynamic roughness height z_o can be calculated via $z_o = k_s \exp(8.5\kappa)$, where $\kappa = 0.4$ is the von Karman constant. We then obtain the roughness function via

$$\Delta U^+ = \frac{1}{\kappa}\log(y^+ - d^+) + B - U^+, \tag{3}$$

where the normalization is via the wall unit, ν/u_τ for length, and u_τ for velocity, and we have denoted with a superscript +; $B = 5$; U^+ at y^+ is obtained from the averaged profile. ΔU^+ is constant within the inertial layer. The roughness function is measured at $y = 0.5\delta_0$. No dependence on the vertical direction is found within $0.3\delta_0 < y < 0.7\delta_0$. Only at the smallest Reynolds numbers can a viscosity effect be discerned in Fig. 2 (left figure) within the roughness layer in between the elements. As a result of the relative insensitivity to Reynolds number when plotted in outer units, the roughness function compared with data from other authors in figure 3 shows good agreement with the expected trends in the fully rough regime. A slight deviation (upward) at low Reynolds number can be observed from figure 3, in good agreement with the Colebrook formula.

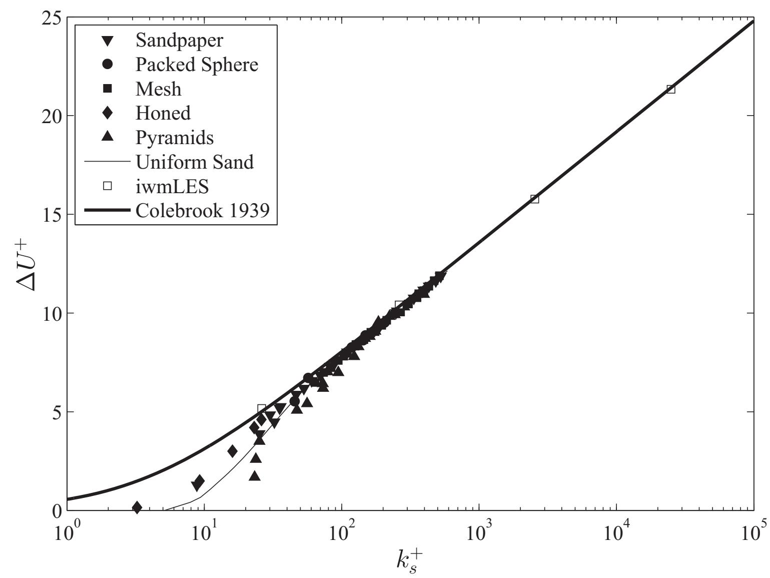

Figure 3. A comparison of the roughness function from this study with data from other authors. The thick line is the 'universal formula' by Ref. 4, $\Delta U^+ = 1/\kappa \log(1 + 0.26k_s^+)$. The sandpaper and mesh data are from Ref. 5, data for packed spheres is from Ref. 6, honed data is from Ref. 7, data for pyramids are from Ref. 8, the data for uniform sand is from Ref. 15.

VI. Conclusion

In this study, a recently proposed integral wall model and a rough wall inflow generation technique are combined in LES of developing turbulent boundary layer over an array of cuboidal roughness of solidity 0.06, at various Reynolds number. We observe that at low Reynolds numbers, an effect can be discerned in the roughness layer. The relative insensitivity to Re at the higher values of Re leads to excellent collapse of the results onto the expected "fully rough" roughness function behavior and good agreement with the 'universal formula' by Ref.,[4] as well as various previous experimental measurements.[5–8]

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