

Finding the Fairest Voting System

A statistical analysis of voting systems to determine which systems are the most likely to jointly satisfy Arrow's conditions.



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Voting Systems

Voting systems are functions that take in voters' preferences and output a ranking of the candidates. Different voting rules can produce **different** winners even with the **same** votes.

Voting Rules

- Plurality
- Borda Count
- Instant Runoff
- Top Two (Runoff)
- Baldwin
- Pairwise Majority
- Imposed Rule

Quantifying Fairness

Fairness Criteria

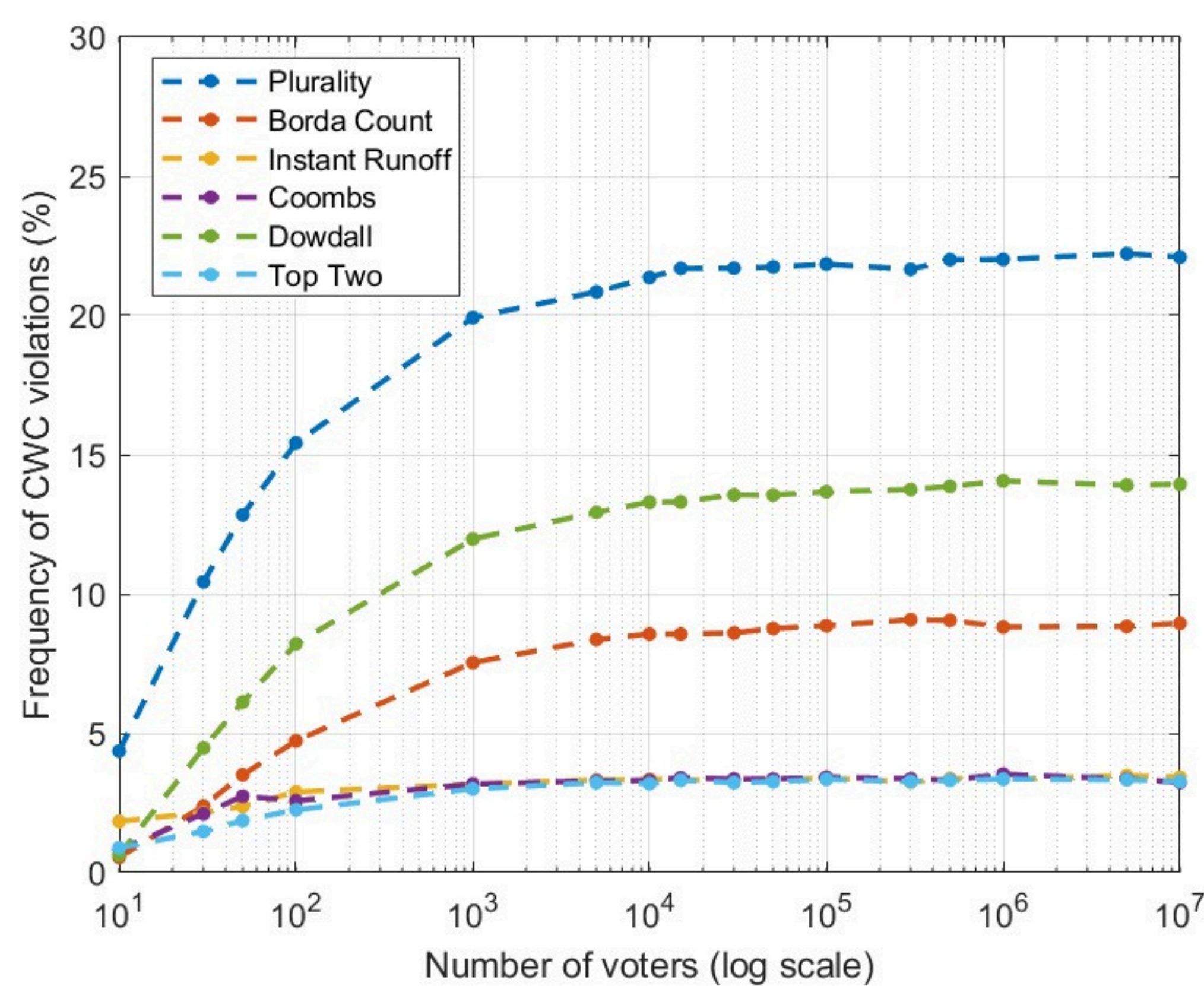
- Unrestricted Domain (UD)
- Non-dictatorship (ND)
- Unanimity (UNM)
- Independence of Irrelevant Alternatives (IIA)
- Transitivity (TR)
- Condorcet Winner Criteria (CWC)

To measure fairness of voting systems, we find the likelihood that they violate fairness criteria. **No voting system satisfies all criteria.**

Experiment

Objective: Find the voting systems that have the highest **Joint-Satisfaction (JS) Frequencies**, likelihood of satisfying all criteria simultaneously.

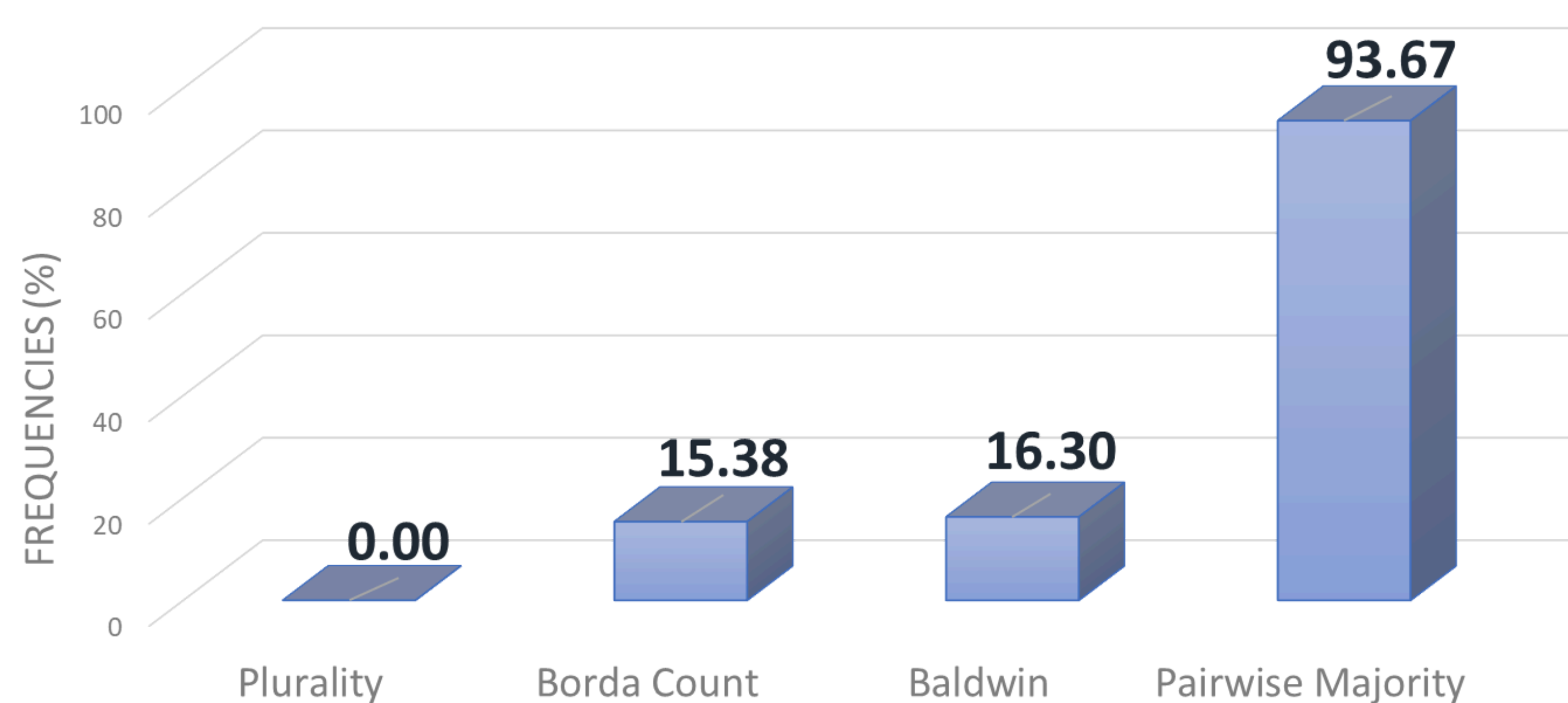
Method: We use Monte Carlo simulations by randomly varying voter preferences and computing numerical probabilities



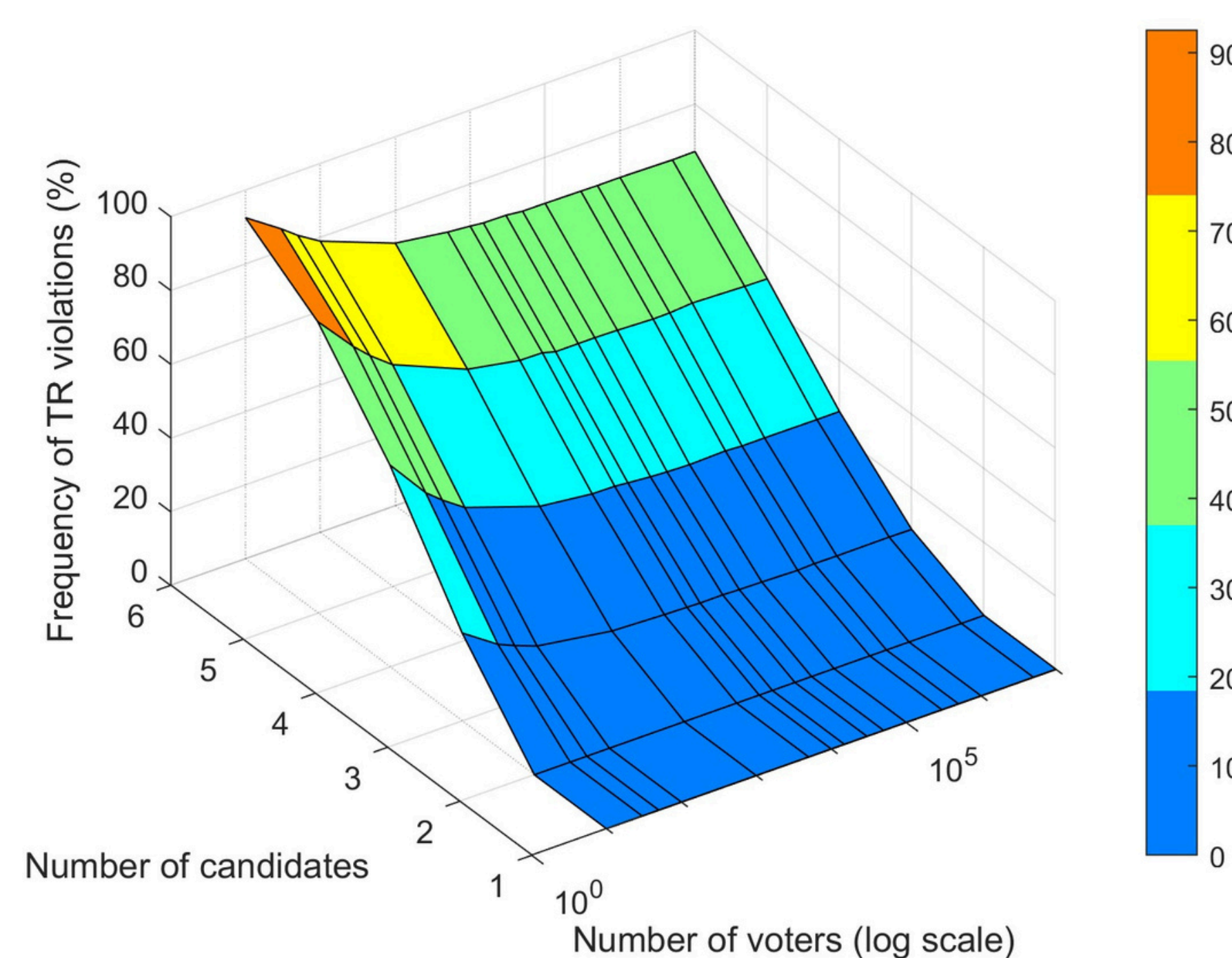
- Voting systems are not all equally likely to violate criteria
- Plurality is **6 times** more likely to violate CWC than Instant Runoff
- UNM, UD, and ND rarely violated

Results

Joint-Satisfaction Frequencies of Voting Systems



- Many systems, including Plurality, have JS Frequencies of 0%
- This is due to high IIA violations: 10 systems **violate IIA** with over 80% probability
- Pairwise Majority only violates TR. Violation frequency graph shown below:



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References

- Arrow, K. J. (1963). Social choice and individual values. (2nd ed.) [First edition published in 1951]. Wiley.
- Dougherty, K. L., & Heckelman, J. C. (2020). The probability of violating arrow's conditions. European Journal of Political Economy, 65, 101936. <https://doi.org/10.1016/j.ejpeleco.2020.101936>
- Hodge, J. K., & Kilma, R. E. (2018). The mathematics of voting and elections: A hands-on approach: Second edition (Vol. 30). American Mathematical Society.