

## Introduction

There is no unified, general framework to **model and query uncertainty propagation across an arbitrary geometric network**. This project bridges **geometry, probability, and simulation**, providing a principled way to understand uncertainty in complex robotic systems.

**Goal:** develop a **general simulation framework** that models and propagates uncertainty through a network of geometric relationships, enabling uncertainty queries between any two nodes.

## Objectives

- Propagate measurement uncertainty analytically through arbitrary kinematic/sensor networks
- Support **open chains, branching paths, multi-path fusion, and loop closure constraints**
- Supports both **1-time** uncertainty (e.g. tolerance) and **multiple time** uncertainty (e.g. noise)
- Validate all analytic results against Monte Carlo simulation

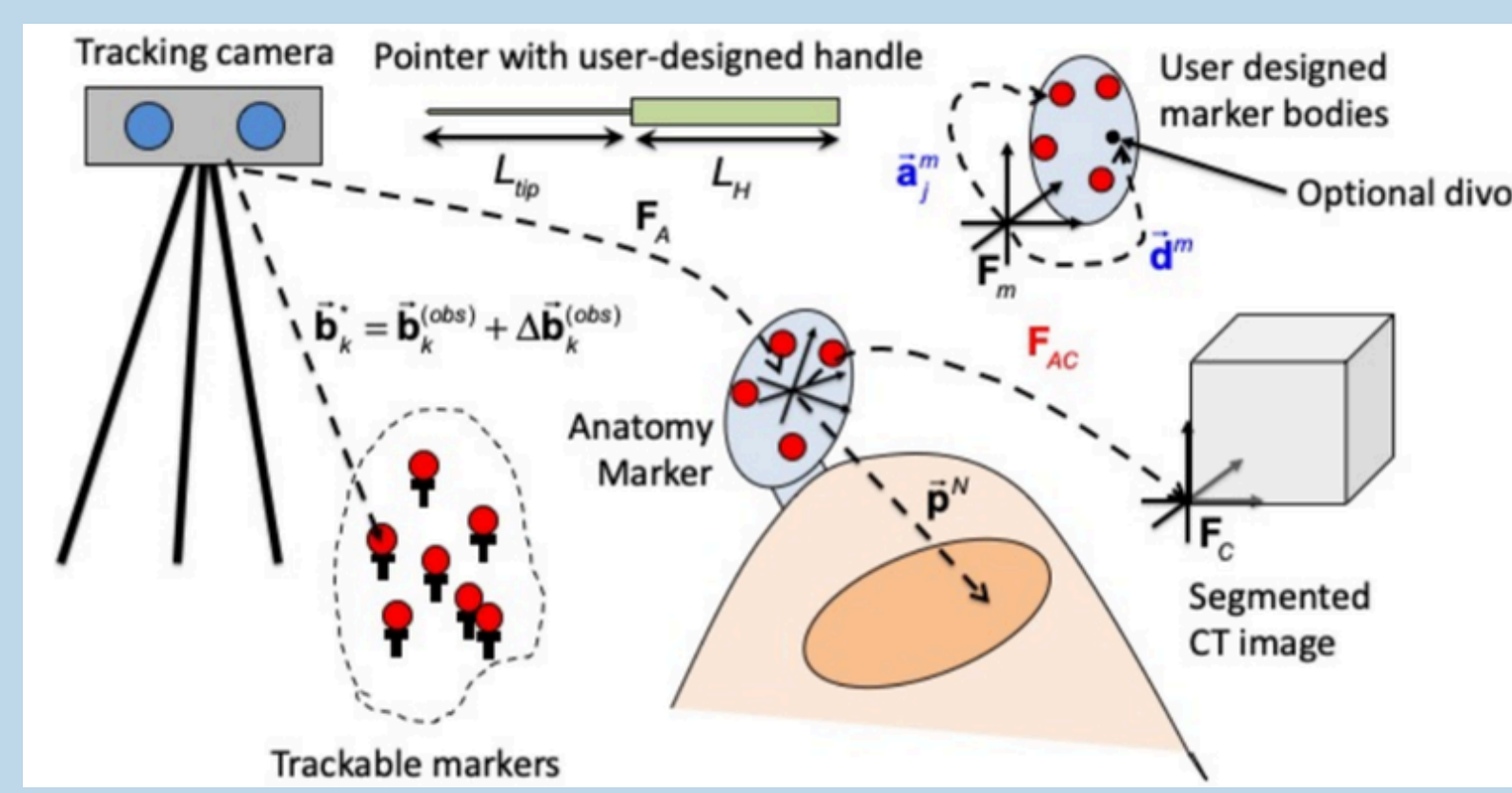
## Methods

- Define uncertainty-aware geometric primitives
- Build composable operators for SE(3) chains
- Represent system as a directed **graph network**
- Support **multi-path Bayesian fusion and loop conditioning**
- Validate analytically and via **Monte Carlo** simulation
- Visualize in **Asynchronous multi-body framework (AMBF)**
- Bridge simulation to real systems via AMBF integration

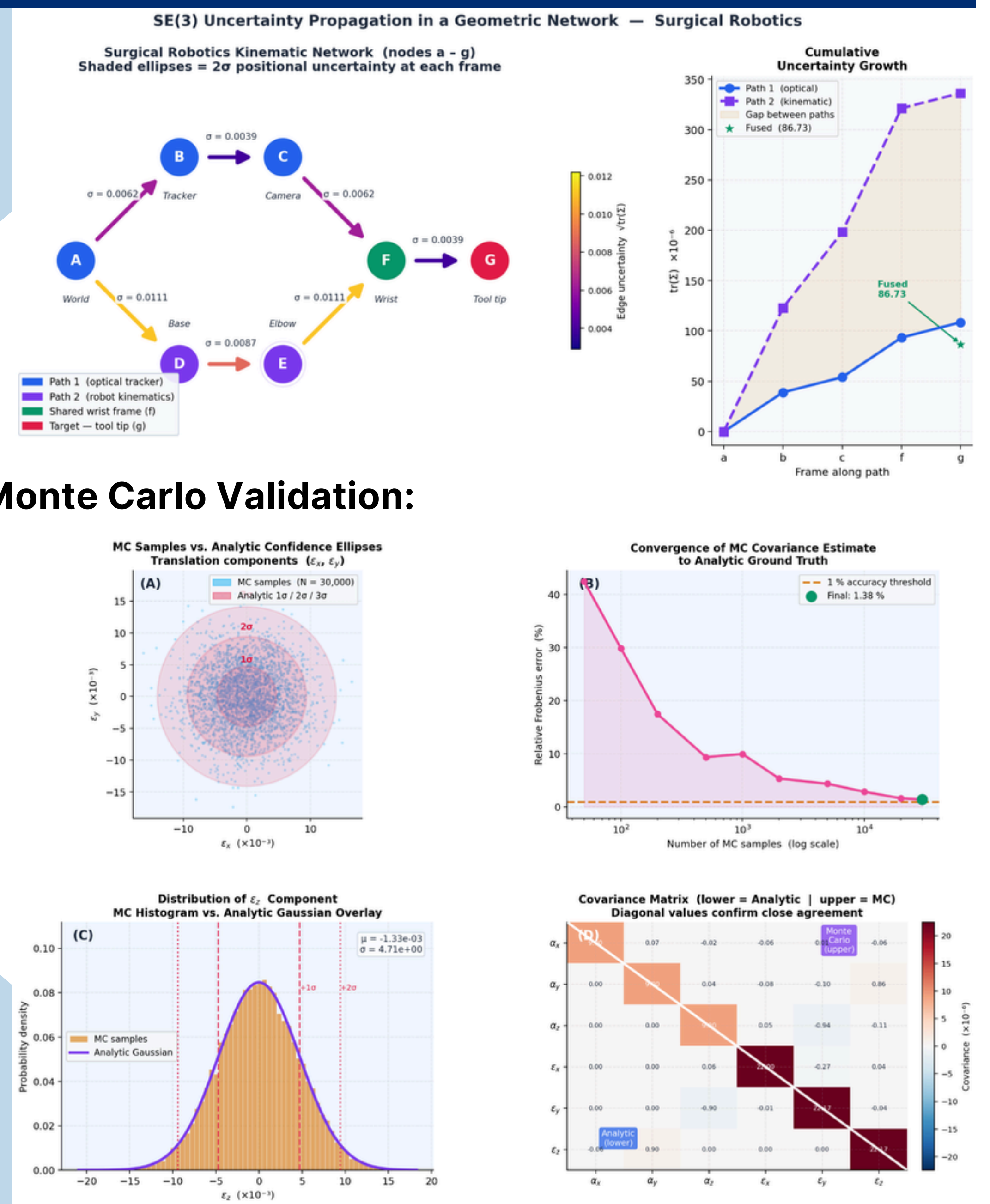
## Results

### 1 Results:

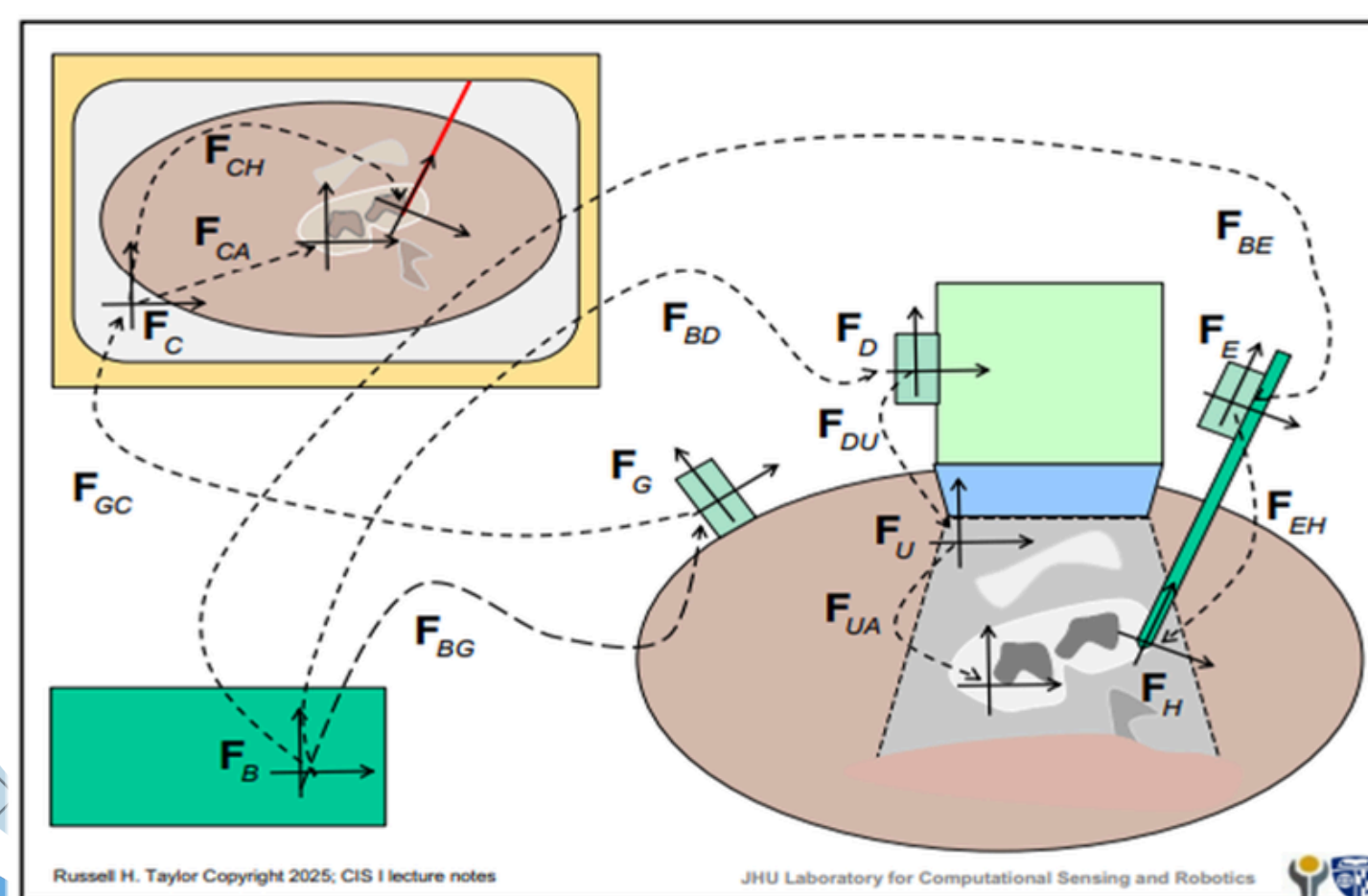
- Analytic covariance matches Monte Carlo (N = 30,000 samples) with **< 2% relative Frobenius error** across open chains, branching networks, and point queries
- Multi-path Bayesian fusion with two equal-uncertainty paths reduces covariance trace by **~50%**
- Closed-loop constraint conditioning reduces uncertainty trace by **> 40%** on diamond network topology
- Mixed observation types (loop + point + distance) all decrease posterior uncertainty monotonically



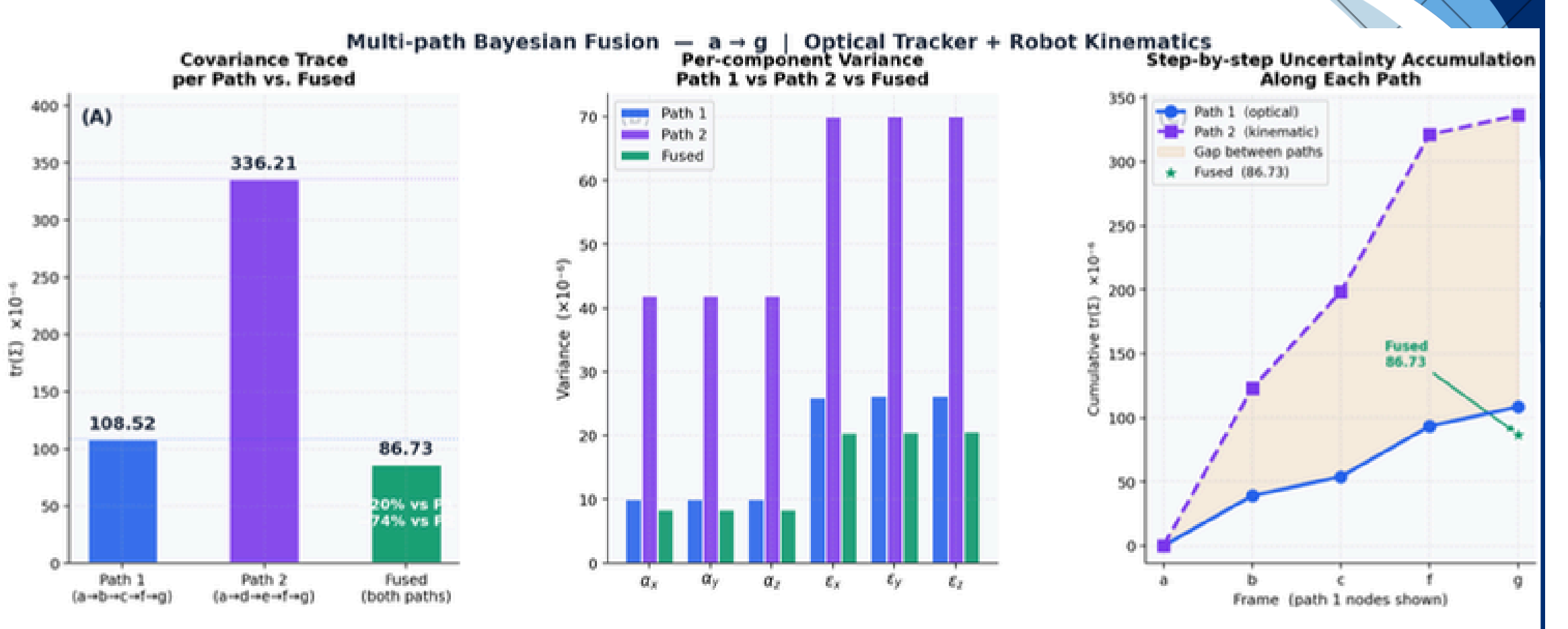
### 2 Monte Carlo Validation:



### 3 System Abstraction:

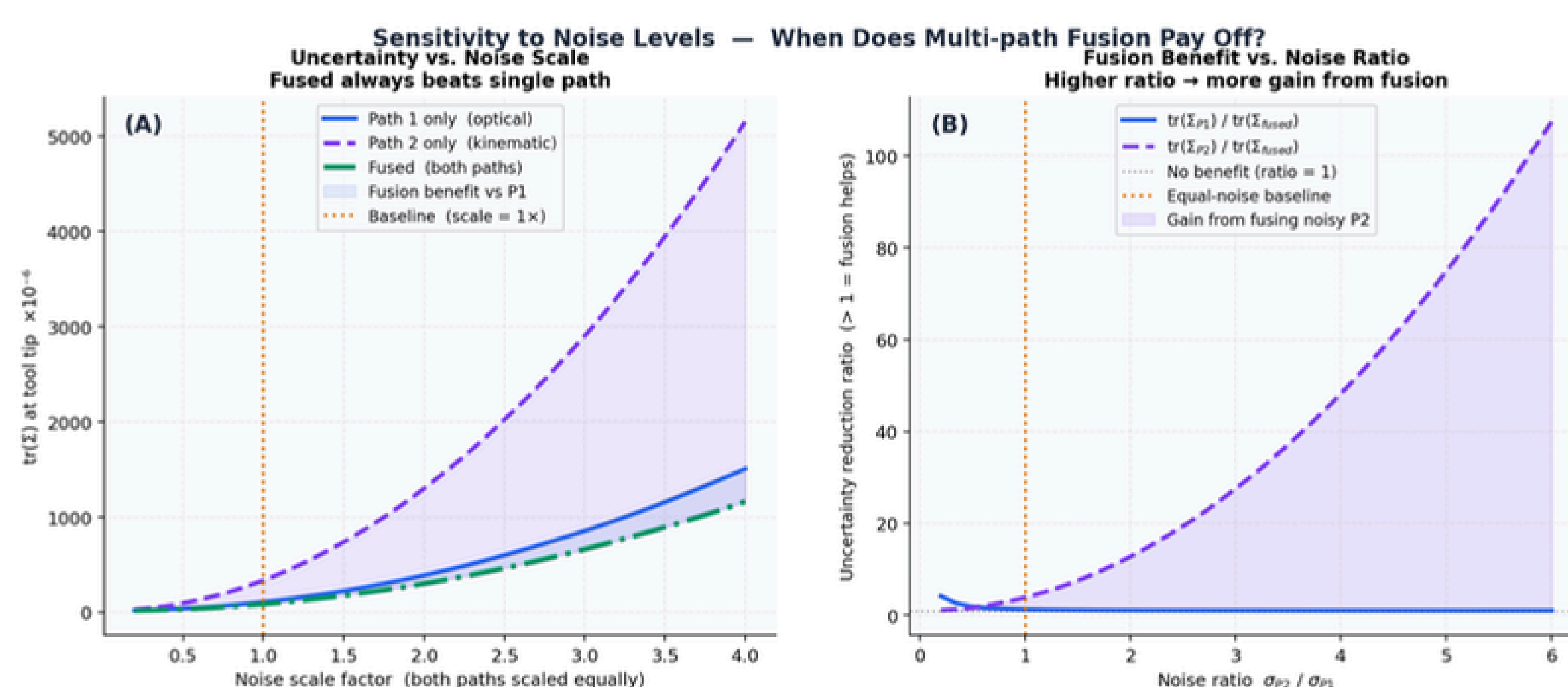


### Multi-path Bayesian Fusion:



### 4 Future Work:

- **AMBF Integration**
  - Integration with AMBF for simulated surgical environment and geometry interaction.
  - **GUI**
  - User interface for interacting with the network, selecting nodes, visualizing paths, and inspecting uncertainty.
- **Full System Integration & Validation**
  - Combine all modules (multi-constraint, observation model, AMBF, visualization) and validate on a surgical use-case scenario.



## Conclusion

- Developed and validated a complete Python framework for **analytic first-order uncertainty propagation** through SE(3) geometric networks
- **Multi-path Bayesian fusion and loop closure conditioning** provide principled tools to reduce uncertainty by exploiting network redundancy — directly applicable to multi-sensor surgical systems
- Monte Carlo validation confirms first-order approximations are accurate (**< 2% error**) for realistic surgical robotics noise scales
- The framework is modular and extensible: supports **open chains, shared hubs, diamond/loop topologies, and arbitrary mixed observation types**