This presentation will provide an introduction of the mathematical background of the lattice Boltzmann equation (LBE). I will first show the derivation of the LBE from the linearized Boltzmann equation. I will discuss the LBE model with multiple-relaxation-times (MRT), of which the lattice Bhatnagar-Gross-Krook (BGK) model with single-relaxation-time is a special case. This allows us to understand the nature and the limitations of the lattice Boltzmann method for applications in computational fluid dynamics (CFD).

In particular, I will discuss the deficiencies of the lattice BGK model. Two examples will be used to demonstrate the efficacy of the lattice Boltzmann method: (1) flow past porous media, and (2) a detailed comparison of the lattice Boltzmann (LB) and the pseudo-spectral (PS) methods for direct numerical simulations (DNS) of the decaying turbulence in a three dimensional periodic cube. Our results show that LBE is an explicit second-order scheme with relatively low numerical dissipation and dispersion.