

Date: December 7th

Time: 11:00 AM

Location: Maryland Hall 110

Speaker: Dr. Eliot Fried

Washington University

Title: *"A Conjectured Hierarchy of Length Scales in a generalization of the Navier-Stokes Equation for Turbulent Fluid Flow"*

Abstract

We present a continuum-mechanical formulation and generalization of the Navier–Stokes-a theory based on a general framework for fluid-dynamical theories with gradient dependencies. Our flow equation involves two additional problem-dependent length scales a and b . The first of these scales enters the theory through the internal kinetic energy, per unit mass, $a^2|\mathbf{D}|^2$, where \mathbf{D} is the symmetric part of the gradient of the filtered velocity. The remaining scale is associated with a dissipative hyperstress which depends linearly on the gradient of the filtered vorticity. When a and b are equal, our flow equation reduces to the Navier–Stokes-a equation. In contrast to the original derivation of the Navier–Stokes-ab equation, which relies on Lagrangian averaging, our formulation delivers boundary conditions. For a confined flow, our boundary conditions involve an additional length scale l characteristic of eddies found near walls. Based on a comparison with direct numerical simulations for fully-developed turbulent flow in a rectangular channel of height $2h$, we find that $a/b \sim Re^{0.470}$ and $l/h \sim Re^{-0.772}$, where Re is the Reynolds number. The first result, which arises as a consequence of identifying the internal kinetic energy with the turbulent kinetic energy, indicates that the choice $a=b$ required to reduce our flow equation to the Navier–Stokes-a equation is likely to be problematic. The second result evinces the classical scaling relation $h/L \sim Re^{-3/4}$ for the ratio of the Kolmogorov microscale h to the integral length scale L . The numerical data also suggests that $l \leq b$. We are therefore led to conjecture a tentative hierarchy, $l \leq b < a$, involving the three length scales entering our theory.