

# Applied mathematics department seminar

September 9th, 2021

SPEAKER: Jim Fill

TITLE: Breaking Multivariate Records

ABSTRACT: For general dimension  $d$ , we identify, with proof, the asymptotic conditional distribution of the number of (Pareto) records broken by an observation given that the observation sets a record.

Fix  $d$ , and let  $\mathcal{K}(d)$  be a random variable with this distribution. We show that the (right) tail of  $\mathcal{K}(d)$  satisfies

$$\mathbb{P}(\mathcal{K}(d) \geq k) \leq \exp \left[ -\Omega \left( k^{(d-1)/(d^2-2)} \right) \right] \quad \text{as } k \rightarrow \infty$$

and

$$\mathbb{P}(\mathcal{K}(d) \geq k) \geq \exp \left[ -O \left( k^{1/(d-1)} \right) \right] \quad \text{as } k \rightarrow \infty.$$

When  $d = 2$ , the description of  $\mathcal{K}(2)$  in terms of a Poisson process agrees with the main result from Fill [*Comb. Probab. Comput.* 30 (2021) 105–123] that the distribution of  $\mathcal{K}(2)$  is Geometric(1/2) with support  $\{0, 1, \dots\}$ .

We show that  $\mathbb{P}(\mathcal{K}(d) \geq 1) = \exp[-\Theta(d)]$  as  $d \rightarrow \infty$ ; in particular,  $\mathcal{K}(d) \rightarrow 0$  in probability as  $d \rightarrow \infty$ .