Nonconvex-Nonconcave Min-Max Optimization with a Small Maximization Domain

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Abstract

We study the problem of finding approximate first-order stationary points in optimization problems of the form $\min_{x \in X} \max_{y \in Y} f(x, y)$, where the sets X, Y are convex and Y is compact. The objective function f is smooth, but assumed neither convex in x nor concave in y. Our approach relies upon replacing the function $f(x, \cdot)$ with its k-th order Taylor approximation (in y) and finding a near-stationary point in the resulting surrogate problem. To guarantee its success, we establish the following result: let the Euclidean diameter of Y be small in terms of the target accuracy ε, namely $O(\varepsilon^{\frac{2}{k+1}})$ for $k \in \mathbb{N}$ and $O(\varepsilon)$ for $k = 0$, with the constant factors controlled by certain regularity parameters of f; then any ε-stationary point in the surrogate problem remains $O(\varepsilon)$-stationary for the initial problem. Moreover, we show that these upper bounds are nearly optimal: the aforementioned reduction provably fails when the diameter of Y is larger. For $0 \leqslant k \leqslant 2$ the surrogate function can be efficiently maximized in y; our general approximation result then leads to efficient algorithms for finding a near-stationary point in nonconvex-nonconcave min-max problems, for which we also provide convergence guarantees.

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