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Ph.D. Candidacy Exam Syllabus

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Research Topics

My current research area of interest is solving mixed integer linear programs (MILP). A useful tool for this is a cutting plane (CP), which has both theoretical interest and practical use. Two methods for generating CP's are outlined in [6]: (1) employing S-free sets and (2) finding minimal inequalities for the infinite group problem, which is essentially a relaxation of an integer program. Dr. Amitabh Basu and I have addressed (1) by generalizing results from [1] that yield explicit CP constructions (see [5]). I look to continue investigating CP's and begin working on other MILP questions.

Exam Topics

- General Topics

  (i) Basic Linear Algebra and Matrix Analysis
  References:
    - [8] Ch. 0-7 – Eigenvalues and eigenvectors; unitary similarity; canonical forms and factorizations; Hermitian matrices; vector and matrix norms; location and perturbation of eigenvalues; positive definite and semidefinite matrices

  (ii) Discrete Geometry
  References:
    - [2] Ch. 6 (Sections 1-5) – Faces of polytopes, polarity, simple polytopes
    - [12] Ch. 1-2 – Polytopes, polyhedra, cones; the face lattice, Farkas' Lemma

  (iii) Convexity theory
  References:
    - [2] Ch. 1-3, 7 – General definitions, Helly's Theorem; extreme points; convex sets in topological vector spaces; lattices
    - [7] Ch. 3,5 – Projection onto convex sets, conical approximations; Sublinearity and support functions

(iv) Combinatorial Optimization
References:

- [11] Ch. 3,5,8 – Max flow/min cut and applications, Ford-Fulkerson algorithm; optimal matchings, Blossom algorithm; matroid axioms and the greedy algorithm

(v) Optimization Theory
References:

- [10] Ch. 7, 11-14, 16-22– Fundamental LP concepts; simplex method, primal-dual method, Fourier-Motzkin elimination, relaxation methods; Khachiyan's method; the ellipsoid method; Fundamental MILP and ILP concepts, estimates in ILP, complexity, total unimodularity, integral polyhedra and total dual integrality
- [11] Ch. 6 – Integrality of polyhedra, integral polytopes, total unimodularity;

- **Specific Topics**

(i) Discrete Geometry
References:

- [12] Ch. 3,4,5,7 – Polytope, representation theorem, graphs of polytopes, the Hirsch conjecture; Steinitz' Theorem for 3-polytopes, Schlegel diagrams for 4-polytopes, fans, zonotopes

(ii) Convexity Theory
References:

- [5] – Lattice free sets, truncated lattices
- [9] – S-free sets

(iii) Optimization Theory
References:

- [3], [5] – Lifting regions and lifting property, minimal cut generating pairs
- [4] – k-dimensional infinite group problem
- [6] – Corner polyhedra and intersection cuts
- [10] Ch. 23-24 – Cutting planes, Chvátal rank, branch and bound methods, the group problem and corner polyhedra, Lagrangean relaxation

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Candidacy Exam Syllabus

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Research Topic. The research topic for this candidacy exam focus on interval (di)graphs, a class of graphs such that each vertex is associated with one or more intervals with intersection of these intervals dictating adjacency of vertices. A more formal definition is given below:

Definition 1. A graph $G = (V, E)$ is an *interval graph* if we can assign to each vertex $v \in V(G)$ an interval I_v on the real line such that for two vertices $v, w \in V(G)$, v is adjacent to w if and only if $I_v \cap I_w \neq \emptyset$ (i.e. the intervals corresponding to v and w overlap).

For interval digraphs, there are two logical extensions of interval graphs we might consider employing:

Definition 2. A directed graph $G = (V, E)$ is an *interval digraph* if we can assign to each vertex $v \in V(G)$ two intervals S_v, R_v on the real line such that for two vertices $v, w \in V(G)$, there is an edge from v to w if and only if $S_v \cap R_w \neq \emptyset$ (i.e. the "send" interval of v and the "receive" interval of w overlap).

Another potential version of interval graphs is the following:

Definition 3. A directed graph $G = (V, E)$ is an *interval digraph* if we can assign to each vertex $v \in V(G)$ an interval I_v on the real line such that for two vertices $v, w \in V(G)$, there is an edge from v to w if and only if one of the following holds:

- $I_v \subseteq I_w$
- $I_w \subseteq I_v$
- $I_v \cap I_w \neq \emptyset$ and the left endpoint of I_v is less than the left endpoint of I_w.

Note that these two definitions are *not* equivalent.

General Topics

- *Graph Theory* (fundamentals of graph theory; graph invariants; random graph models (eg. Erdös-Rényi); classes of graphs; hereditary properties of graphs; forbidden subgraphs; threshold functions)

  References: [11] Chapters 1-7; [2]; [5]

- *Random Methods* (discrete random variables; Markov, Chebyshev, and Chernoff bounds; Probabilistic Method; Lovasz Local Lemma)

  References: [1] Chapters 1-4

Specific Topics

- *Interval digraphs*

  References: [3]; [6]; [7]; [8]; [9]

- *Efficient Graph Representations* (asymptotics; local representation; representations defined by forbidden subgraphs)

  References: [10] Chapters 1, 2, 8, 13.2; [4

References

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Ph.D. Candidacy Exam Syllabus

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Research Topics:

I am interested in researching the theory of wavelets and signal processing. In particular, I intend to study the construction of wavelets and the analysis of their smoothness and representation power, especially in multiple dimensions, which is where most open questions in wavelet analysis lie. The construction of nonseparable multidimensional wavelets (i.e., multidimensional wavelets which are not the tensor product of univariate wavelets) is highly non-trivial, and the analysis of their properties also becomes substantially more complicated than in the univariate or separable cases. However, these questions are extremely important in an age when multidimensional signals such as images and videos are being sent and received at an ever-increasing rate, and the volume of high-dimensional data generated in experiments requires efficient representations to make possible their analysis, denoising, and transmission.

Exam Topics:

General Topics:

- **Analysis and Measure Theory:** Basic definitions and properties of the integral; notions of convergence for a sequence of measures; definitions, properties, and convergence of the Fourier transform for finite measures ([3] Sections 3.1-3.2, and chapters 4 and 6). Lebesgue integration on $\mathbb{R}^n$; differentiation and approximations to the identity; basic Hilbert space theory and examples ([17] Chapters 2 and 4, and sections 3.1-3.3 and 5.1-5.2; problems from 3.5-3.6 related to sections 3.1-3.3, and problems from 5.5-5.6 relating to sections 5.1-5.2).
- **Algebra:** Basic properties of groups, rings, and modules ([14] Sections 1.1-1.10, 1.12, 3.1-3.6, 3.10, 4.1-4.2, and chapter 2).
- **Matrix Analysis and Linear Algebra:** Various basic facts about matrices and linear algebra; eigenvectors, eigenvalues, and similarity; unitary similarity and unitary equivalence; canonical forms for similarity and triangular factorizations; Hermitian matrices; norms; location and perturbation of eigenvalues ([7] Chapters 0-6 except sections 2.7 and 4.4-4.6).
- **Functional Analysis:** Basic definitions and facts about Sobolev spaces ([6] Sections 5.1-5.8; problems from 5.10 relating to sections 5.1-5.8). Basic properties of Fourier series; convergence of Fourier series; the Fourier transform on $\mathbb{R}$ and $\mathbb{R}^d$ ([16] Chapters 2, 3, and 5, and sections 6.1-6.2; problems in 6.6-6.7 related to 6.1-6.2). L^p and Banach spaces; L^p spaces in harmonic analysis; generalized functions (distributions) ([18] Chapters 1-3).

Specific Topics:

- **Wavelet Theory:** The continuous wavelet transform; discrete wavelet transforms and frames; time-frequency density and orthonormal bases; orthonormal bases of wavelets and multiresolution analysis; compactly supported wavelets and smoothness estimates; characterization of functional spaces ([5] Chapters 2-7 and 9). Multiresolution approximations of $L^2(\mathbb{R}^n)$ and orthonormal wavelets ([15] Chapter 2 except sections 2.9-2.11, chapter 3); Signals, samples, and time-invariance; Shannon sampling; filter banks, perfect reconstruction, and the polyphase representation; orthogonal filter banks; wavelet filter banks; approximation accuracy, cascade algorithm, smoothness; multidimensional wavelets; filter bank design methods ([19] Sections 2.1-2.2, 4.1-4.4, 7.1-7.3, 9.5, and 10.1-10.4, and chapters 5 and 6).

- **Gröbner Bases:** Basic theory and applications of Gröbner bases, including applications of Gröbner bases to modules ([1] Chapters 1-3 except 2.6-8 and 3.9-10). Basic algebraic geometry and commutative algebra; Gröbner bases; elimination theory ([4] Chapters 1-4).

- **Wavelet Construction:** The coset sum method [12]. Prime coset sum method [11]. Effortless critical representation of the Laplacian pyramid [8]. Wavelet filter bank design using Quillen-Suslin theorem [10]. Scalable frames and wavelet filter banks from the Laplacian pyramid [9]. Construction of tight wavelet frames using real algebraic geometry [2]. Construction of tight wavelet frames from filters satisfying the sub-QMF condition using some theory of positive polynomials [13].

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Research Topics

When searching or sorting algorithms are applied to keys (a term used in computer science for the identifying labels of records) that are represented as bit strings, we can quantify the efficiency of the algorithms not only in terms of the number of key comparisons required by the algorithms but also in terms of the number of bit comparisons. Some of the standard algorithms for searching and sorting have been analyzed with respect to key comparisons but not with respect to bit comparisons. Professor James Allen Fill and I are currently investigating the bit complexity of Quickselect; we examine the expected number of bit comparisons required by Quickselect when (for example) the bits are assumed to result from Bernoulli trials. We also intend to embark on a research project on Markov chains.

Exam Topics

General Topics

• **Probability** Basic concepts in probability.
References: [1].

• **Stochastic Processes** Poisson processes, Markov chains, martingales, Brownian motion, stochastic calculus.
References: [2], [3].

• **Analysis of Algorithms** Recurrences (substitution, recursion-tree, and master methods), probablistic analysis and randomized algorithms, sorting and selection problems, heaps, elementary data structures, hash tables, binary search trees, red-black trees.
References: [4] Chapters 1 through 13.

Specific Topics

• **Probability Theory** Modes of convergence, the central limit theorem, laws of large numbers, characteristic functions.
References: [5] Chapters 1 through 5.

• **Ergodic Markov Chains** Convergence to stationarity, Markov chain Monte Carlo, Glauber dynamics, perfect sampling.
References: [6].

• **Complex Analysis** Holomorphic and meromorphic functions, Cauchy's theorem and integral formulae, the residue theorem, Rice's method, the gamma and zeta functions.
References: [7] Chapters 1 through 6; [8] Chapter 6.

• **Combinatorics** Recurrence relations and generating functions.
References: [9] Chapter 7.

• **Quickselect and Quicksort** Expected numbers of key and bit comparisons required by Quickselect and Quicksort.
References: [10].

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Research Topic

My area of current research interest concerns **Markov chains.** I am interested in their applications to perfect sampling algorithms, and in the methods used to bound mixing times for these chains. Specifically, Fill's perfect sampling algorithm, originally limited to monotone Markov chains, was generalized by Fill et al. [6] to be able to handle a wider variety of chains; in the generalization, the user can choose the initial state. Currently, there are only a few examples [4] where it is known what constitutes a "good" or "bad" starting state for the algorithm; there are no general guidelines, rigorous or even heuristic, available. Developing such guidelines is a project I intend to pursue. Also I plan on pursuing a research project in the analysis of algorithms.

Exam topics

General Topics

• **Probability** Basic concepts in probability.
Reference: [8].

• **Stochastic Processes** Stochastic processes, Markov chains.
Reference: [9] Chapters 1-5.

Specific Topics

• **Probability Theory** Modes of convergence, laws of large numbers, characteristic functions, conditional probability and martingales.
Reference: [2] Chapters 2 through 6 and Chapter 9.

• **Reversible Markov Chains** Spectral representation, extremal characterizations of relaxation time and other Markov chain parameters, coupling, distinguished paths method, the comparison method.
Reference: [1] Chapters 3, 4 (Section 4.3 only), 4-3, and 8 (Section 1 only).

• **Strong Stationary Duality** Strong stationary times, strong stationary duality.
Reference: [3].

• **Perfect Sampling Algorithms** Coupling from the past, Fill's perfect sampling rejection algorithm.
References: [7], [5].

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Ph.D. Candidacy Exam Syllabus

Research Topic: Spectral Graph Theory

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Research Topic

The research problem I have investigated involves representing a graph by a set of vectors and their corresponding dot products. In its most basic setting, the problem can be stated loosely as follows: given a specific graph, find a set of vectors whose dot products generate this graph. An equivalent problem is to find an appropriate diagonal matrix to add to the adjacency matrix of the graph to get a resultant matrix that is both positive semidefinite and of lowest possible rank. Thus, we are interested in shifting the eigenvalues of the adjacency matrix in such a way that the eigenvalues are all nonnegative and the number of zero eigenvalues is maximized. This specific topic falls under the more general problem of examining the eigenvalues of a graph and their connections to invariants of the graph, which is a main focus of spectral graph theory and the topic for this exam.

General Material

Graph Theory.

Fundamental graph concepts; classes of graphs and related properties; matrices associated with graphs; eigenvalues of graphs.

References: [5] chapters 8-9; [13] in general, specifically chapter 8 (section 6).

Matrix Analysis.

Eigenvalues and eigenvectors; symmetric matrices; positive semidefinite matrices; singular value decomposition; interlacing of eigenvalues; Perron-Frobenius theory for irreducible nonnegative matrices.

References: [8] chapters 1, 4, 7, 8 (section 4).

Specific Material

Basic properties of spectrum of a graph.

Different types of graph spectra (adjacency, Laplacian, normalized versions); basic bounds.

References: [1] section 2; [4] chapter 1; [7].

Structural properties and invariants of graph related to adjacency matrix spectrum.

Bipartite graphs; regular graphs; strongly regular graphs; line graphs.

References: [1] section 3; [3]; [4] chapter 3; [5] chapters 10, 12.

Characterizations of graphs by means of adjacency matrix spectra.

Families of non-isomorphic cospectral graphs; graphs determined by their spectra.

References: [4] chapter 6; [12].

Structural properties and invariants of graph related to Laplacian spectrum.

Characteristics and bounds of Laplacian spectrum; Matrix-Tree Theorem, eigenvalues and random walks on graphs; isoperimetric problems (Cheeger's inequality); diameters and eigenvalues; algebraic connectivity and spectral graph partitioning.

References: [1] section 6; [2] chapters 1-3; [4] chapter 9 (section 3); [5] chapter 13; [6]; [9]; [10].

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Candidacy Exam Syllabus [redacted]

Research Topic: My area of interest falls within operations research: in particular a two stage optimization problem of aggregation and schedule creation. In this problem there is a digraph D, whose vertices must be clustered into parts that shall become the vertices of a new digraph D'. The challenge is to group the original vertices in a meaningful fashion, while assigning directed edges to the resultant graph to guide scheduling the new vertices optimally. This general problem has various applications, such as job shop scheduling when the initial jobs must be formed from smaller sub-jobs, and university curriculum creation, where courses in the curriculum are to be formed from basic topics. An RAship on which I have been working, focusing on curriculum generation, was the primary impetus for this research.

I)	**Integer Programming Modeling**		
	i)	General modeling techniques	3: 10.2
	ii)	Guidelines for strong formulations	3: 10.2
	iii)	IP duality	3: 11.4
II)	**Solution Techniques**		
	i)	Cutting plane algorithms	7: II.4.3; 8: 14.1-14.3
	ii)	Branch and bound	2: 5.5; 3: 11.2; 8: 18.1-18.4
	iii)	Dynamic programming	1: 3; 3: 11.3; 8: 18.6
III)	**Computational Complexity**		
	i)	Definition of computability, P, NP, NP complete, Co-NP	5: 7.1; 8: 8.1, 15.3, 16.1
	ii)	The size of an instance	8: 8.3
	iii)	Polynomial time reductions	8: 8.2
	iv)	Cook's Theorem	5: 2.6; 8: 15.4-15.5
	v)	Known NP complete problems	5: 3.1; 8: 15.6-15.7
	vi)	'Easier' cases of NP complete problems	8: 16.3
IV)	**Approximation Techniques**		
	i)	Approximation Schemes	8: 17.3
	ii)	Pseudo-polynomial algorithms and strong NP completeness	5: 4.2; 8: 16.2
	iii)	IP approximation algorithms	3: 11.5
	iv)	Simulated annealing	3: 11.7
	v)	Lagrangean relaxation	2: 5.5
	vi)	Benders' decomposition	7: II.3.7, II.5.4
V)	**Relevant Problems**		
	i)	Bin packing	6
	ii)	Partitioning into Triangles	5: 3.2

iii)	Precedence Constrained Scheduling	5: 4.1
iv)	Curriculum generation optimization	4

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Ph.D. Candidacy Exam Syllabus for [redacted]

Research Topic.

My general area of research is stochastic optimization, specifically stochastic approximation. Since the introduction of this method by Robbins and Monro (1951), there has been a great deal of theoretical work on techniques, convergence, and the asymptotic behavior of these methods. In fact, most of what is known about these procedures is known only asymptotically. My research attempts to illuminate the small-sample behavior of these methods. Specifically, I am studying implementable methods to stop stochastic approximation procedures and determine the level of statistical confidence in the results.

Exam Topics.

<u>General Knowledge.</u>

Iterative Optimization.
Gradient and quasi-gradient methods, convergence and stopping rules, penalty methods.
References: [1] chapters 7-10; [4] chapters 1, 2; [12] chapters 1, 15; [14].

Probability.
Functions of random variables, expectation and conditional expectation, convergence of sequences of random variables, limit theorems.
References: [3] sections 20-22, 25, 33, 34; [5] chapters 2-5, 7-9; [12] appendix C.

Statistics.
Maximum likelihood estimation, confidence intervals, hypothesis testing, multivariate random variables, covariance.
References: [2] chapters 3, 5-6; [3] section 20; [12] chapters 12, 13, appendix B.

<u>Specific Knowledge.</u>

Stochastic Iterative Optimization.
Linear models of prediction and estimation, stability analysis (Lyapunov's method), convergence.
References: [4] chapters 4-6; [7] chapters 4-6; [14].

Stochastic Approximation (SA).
Procedures of Robbins-Monro, Keifer-Wolfowitz, and Ruppert-Polyak, iterate averaging, convergence, asymptotic behavior.
References: [6]; [7] chapters 1, 10, 11; [8]; [10]; [11]; [12] chapters 4-6.

Stopping.
Martingales, definitions and theory of stopping rules, stopping stochastic algorithms, stopping SA algorithms, stepsize rules.
References: [3] section 35; [5] chapter 9, sections 1-4; [9]; [13]; [15].

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Research Topics

My main research interest is the study of random discrete structures. Two important problems are (1) the development of efficient algorithms to randomly sample combinatorial objects and (2) properties of large random structures. Each of these problems is interesting mathematically as well as from an applied perspective. Efficient sampling algorithms, for example rapidly mixing Markov chains and the Randomness Recycler, can be used in Monte Carlo schemes for hypothesis testing or to address problem (2). Solutions to problem (2) are often in the form of probabilistic limit theorems; in fact, limit theorems can also provide an alternative to Monte Carlo simulation. Furthermore, limit theorems can be predictive when the structures model phenomena in science and engineering. For example, limit theorems can describe the the structure of a random network or the behavior of an algorithm.

Exam Topics

General Topics

• **Basic Probability** Elementary concepts in probability
References: [Ros06]

• **Markov Chains** Markov chains, Poisson processes
References: [LPW09] Chapter 1, [Ros96] Chapter 2, [HJ85] Chapter 8

• **Combinatorics and Graph Theory**
References: [Mer03] Chapters 1 and 2, [Wes96] Chapters 1, 2, and 5

Specific Topics

• **Probability Theory** Modes of convergence, laws of large numbers, central limit theorem
References: [Chu01] Chapters 2-7

• **Markov Chain Mixing Times** Gibbs sampler, discrepancy from stationarity, coupling, strong stationary times
References: [LPW09] Chapters 3-6

• **Perfect Sampling** Coupling from the Past, FMMR, Randomness Recycler
References: [PW96], [Fil98], [FMMR00], [FH00]

References

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Research Topic

My area of current research interest concerns **Markov chains.** Specifically, I'm interested in **mixing time analysis** for specific Gibbs sampler chains described in references [6] and [7].

Diaconis, Khare, and Saloff-Coste [6] bound the mixing time of two types of bivariate Gibbs samplers by making effective use of stochastic monotonicity [8] and a monotone eigenfunction for one of the two marginal chains.

I intend to compare the random-scan and systematic-scan versions of Gibbs samplers, carry out the analysis beyond the determination of eigenvalues and eigenfunctions (in specific instances), and also further investigate the Diaconis *et al.* coupling/monotonicity arguments in [6] and [7] at least for discrete state spaces, with the aim of expanding arguments there to Gibbs samplers with three or more components.

Exam topics

General Topics

• **Probability** Basic concepts in probability.
Reference: [1].

• **Stochastic Processes** Stochastic processes, Markov chains, Martingales, Brownian motion.
Reference: [2].

• **Matrix Analysis** Nonnegative matrices.
Reference: [3] Chapter 8.

Specific Topics

• **Probability Theory** Modes of convergence, Central limit theorem, Laws of large numbers, Characteristic functions, Conditional probability, Martingales.
References: [4] Chapters 1 through 6.

• **Ergodic Markov Chains** Convergence to stationarity, Markov chain Monte Carlo, Glauber dynamics, Perfect sampling.
References: [5] Chapters 1 through 7.

• **Gibbs Sampler** Convergence to stationarity of families of examples, Quantitative analysis.
References: [6], [7].

• **Monotonicity for Families of Probability Measures** Stochastic monotonicity and realizable monotonicity.
Reference: [8].

References

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