Optimizing the Scaling Parameter for $\rho/\mu$
Approximation Based Control Synthesis

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Abstract—We revisit the problem of designing a full-state feedback controller for a deterministic finite state machine, so as to maximize a performance parameter $R$ while simultaneously ensuring that the closed loop system satisfies a given performance objective involving a positive scaling parameter $\tau$. Under some additional assumptions, we show that the problem of choosing $\tau$ to optimize $R$ in closed loop admits an analytical solution. We demonstrate the use of this approach via a numerical example, showing substantial computational savings over the existing sampling based method. We also provide an intuitive, graph-theoretic interpretation of our result.

I. INTRODUCTION

The past decades have witnessed increasing research interest in using finite state machines as approximate models of more complex hybrid systems. Such models are easier to work with for analysis and control design. In order for the approximate models to be useful in this context, they need to be constructed so as to allow engineers to quantify the performance of a controller designed for the lower complexity model and then implemented in feedback with the original system. Several distinct and complementary notions of approximation have been developed, including qualitative models [1], [2], [3], [4], [5], simulation and bisimulation abstractions [6], [7], [8], [9], and $\rho/\mu$ approximations [10], [11], [12], [13], [14], [15].

Depending on the notion of approximation used, the process of control design can vary, as can the resulting controllers. For instance, controller synthesis for qualitative models can be formulated as a supervisory control problem, addressed using the Ramadge-Wonham framework [16]. Controller synthesis using simulation and bisimulation abstractions is a two step procedure in which a finite state supervisory controller is first designed and then subsequently refined into a certified hybrid controller [17]. In the $\rho/\mu$ approximation framework, control synthesis reduces to designing a full state feedback controller for a deterministic finite state machine approximate model, termed a $\rho/\mu$ approximation [13].

In particular, in [12], [11] the authors provide a systematic approach for using $\rho/\mu$ approximations to construct a finite state stabilizing controller that maximizes the provable rate of convergence $R$, for switched second order homogenous systems. Consequently, control synthesis reduces to designing a controller that maximizes the value of $R$ for the $\rho/\mu$ approximation while simultaneously meeting a closed loop performance objective described as a gain condition involving $R$ and a positive scaling parameter $\tau$. While authors also provide a value iterative algorithm to design the controller for fixed values of $\tau$ and $R$, the question of how to optimize over the parameter space remains minimally explored. Indeed [12], [11], a brute force computational solution was used, effectively sampling the parameter space in search of suboptimal values.

The main contributions of this paper are:

1) Under additional assumptions on the system inputs, we propose and derive an analytical solution for the problem of picking the scaling parameter.

2) We provide a network based intuitive interpretation of our approach.

3) We implement our solution in an illustrative numerical example, thereby demonstrating a dramatic improvement in computation time relative to the sampling based method.

Organization: We present the problem statement in Section II. We explain the existing solution in Section III. We present analysis and propose a more computationally efficient solution in Section IV. We present an illustrative example in Section VI-B.

Notation: $\mathbb{Z}_+$, $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of nonnegative integers, reals and non-negative reals, respectively. Given sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A}^{\mathbb{Z}_+}$ denotes the set of all infinite sequences over $\mathcal{A}$ (indexed by $\mathbb{Z}_+$). For a sequence in $\mathcal{A}^{\mathbb{Z}_+}$, $a_t$ denotes its $t^{th}$ component while $a$ and $\{a_t\}$ are used interchangeably to denote the sequence.

II. PROBLEM SETUP

Consider a deterministic finite state machine (DFM) $M$ described by

$$
q_{t+1} = f(q_t, u_t, w_t),
$$

$$
\hat{v}_t = h(q_t, u_t)
$$

where $t \in \mathbb{Z}_+$, $q_t \in \mathcal{Q}$, $u_t \in \mathcal{U}$, $w_t \in \mathcal{W}$ and $\hat{v}_t \in \mathcal{V}$. The finite sets $\mathcal{Q}, \mathcal{U}, \mathcal{W}$ and $\mathcal{V}$ are the state set, the control input alphabet, the exogenous input alphabet and the performance output alphabet, respectively. Functions $f : \mathcal{Q} \times \mathcal{U} \times \mathcal{W} \rightarrow \mathcal{Q}$ and $h : \mathcal{Q} \times \mathcal{U} \rightarrow \mathcal{V}$ are given. $q_0 \in \mathcal{Q}$ is a given initial state.

The goal is to design a full-state feedback controller $\phi : \mathcal{Q} \rightarrow \mathcal{U}$ so as to maximize the value of performance parameter $R \geq 0$ such that the closed-loop system shown in
Figure 1 satisfies the inequality
\[
\inf_{T \geq 0} \sum_{t=0}^{T} -R - \dot{v}_t + \tau \left( \mu_\Delta (w_t) - \gamma \rho_\Delta (\varphi(q_t)) \right) > -\infty \quad (2)
\]
for some scaling parameter \( \tau > 0 \), \( \gamma \geq 0 \) is a given constant and \( \mu_\Delta : \mathcal{W} \to \mathbb{R}_+ \), \( \rho_\Delta : \mathcal{U} \to \mathbb{R}_+ \) are two given functions.

In particular cases of interest, the exogenous input is due to the approximation process and resulting modeling uncertainty. As such, we have an additional assumption that input sequence pairs \((u, w) \in \mathcal{U}^\mathbb{Z}_+ \times \mathcal{W}^\mathbb{Z}_+ \) satisfy
\[
\inf_{T \geq 0} \sum_{t=0}^{T} \gamma \rho_\Delta (u_t) - \mu_\Delta (w_t) > -\infty. \quad (3)
\]

III. OVERVIEW OF THE EXISTING SOLUTION

The existing solution proposed in [12] systematically samples the parameter space in search of a suboptimal value of \( R \) for which control design is possible. It relies on the fact that for a given DFM and for a fixed choice of parameters \( \tau \) and \( R \), proving or disproving the existence of a state feedback control law \( \varphi : \mathcal{Q} \to \mathcal{U} \) such that the closed loop system satisfies (2) is a readily solvable problem. Indeed, consider the following result:

**Theorem 1.** (Adapted from Theorem 4 in [12]) Consider a DFM \( M \) as in (1) and let \( \sigma : \mathcal{Q} \times \mathcal{U} \times \mathcal{W} \to \mathbb{R} \) be given. The following three statements are equivalent:

a) There exists a \( \varphi : \mathcal{Q} \to \mathcal{U} \) such that the closed loop system \((M, \varphi)\) satisfies
\[
\inf_{T \geq 0} \sum_{t=0}^{T} \sigma(q_t, \varphi(q_t), w_t) > -\infty. \quad (4)
\]

b) There exists a function \( J : \mathcal{Q} \to \mathbb{R}_+ \) such that the inequality
\[
J(q) \geq \mathbb{T}(J(q))
\]
holds for any \( q \in \mathcal{Q} \), for \( \mathbb{T} : \mathcal{R}^\mathcal{Q} \to \mathbb{R}^\mathcal{Q} \) defined by
\[
\mathbb{T}(J(q)) = \min_{u \in \mathcal{U}} \max_{w \in \mathcal{W}} \{-\sigma(q, u, w) + J(f(q, u, w))\}.
\]

c) The sequence of functions \( J_k : \mathcal{Q} \to \mathbb{R}_+ \), \( k \in \mathbb{Z}_+ \), defined recursively by
\[
J_0 = 0 \quad \text{and} \quad J_{k+1} = \max\{0, \mathbb{T}(J_k)\}
\]
converges.

In particular, for a fixed value of \( \tau > 0 \) and \( R \geq 0 \), one can use Theorem 1 to prove or disprove the existence of a state feedback control law \( \varphi : \mathcal{Q} \to \mathcal{U} \) meeting the performance objective defined by
\[
\sigma(q, u, w) = -R - h(q, u) + \tau \left( \mu_\Delta (w) - \gamma \rho_\Delta (u) \right).
\]

The sampling approach presented in [12] can thus be summarized as follows: First, one computes the range of values of \( \tau \) for which (2) can be met for \( R = 0 \). Then, one samples this range to compute the largest value of \( R \) at each sampling, with the largest of those being the (suboptimal) guaranteed rate of convergence.

This sampling approach requires solving a minimax problem at every sampling point, resulting in a computational cost that grows with the number of samples. This motivates the need for an analytical solution to the question: How should one choose the scaling parameter \( \tau \) to optimize \( R \) to optimize \( R \)?

IV. MAIN RESULTS

We establish a set of observations leading to an analytical solution to the question of picking the scaling parameter \( \tau \) to optimize \( R \). For convenience, we begin by defining a feasible pair of parameters:

**Definition 1.** Given \( M \) as in (1) and a performance objective (2). The pair \((\tau, R)\), \( \tau > 0 \) and \( R \geq 0 \), is feasible if there exists a \( \varphi : \mathcal{Q} \to \mathcal{U} \) such that the closed loop system \((M, \varphi)\) satisfies (2).

For convenience, we will use \( \sigma_{\tau, R} \) to denote the summand in (2) for a fixed value of \( \tau \) and \( R \). Specifically, given a choice of \( \tau > 0 \) and \( R \geq 0 \), \( \sigma_{\tau, R} : \mathcal{Q} \times \mathcal{U} \times \mathcal{W} \to \mathbb{R} \) is defined by
\[
\sigma_{\tau, R}(q, u, w) = -R - h(q, u) + \tau \left( \mu_\Delta (w) - \gamma \rho_\Delta (u) \right). \quad (5)
\]

**Proposition 1.** Given \( M \) as in (1), a performance objective (2) and a choice of \( \tau > 0 \). If \((\tau, R_1)\) is feasible for some \( R_1 \geq 0 \), then \((\tau, R_2)\) is feasible for any \( R_2 \leq R_1 \).

**Proof.** When \( R_2 \leq R_1 \), we have
\[
\sigma_{\tau, R_2}(q, u, w) \geq \sigma_{\tau, R_1}(q, u, w), \forall q, u, w. \quad (6)
\]

Let \( \{q_t\} \) be the state sequence of \( M \) starting from initial state \( q_0 = q_0 \) under input sequences \( \{u_t\} \) and \( \{w_t\} \). It follows from (6) that
\[
\sum_{t=0}^{T} \sigma_{\tau, R_2}(q_t, u_t, w_t) \geq \sum_{t=0}^{T} \sigma_{\tau, R_1}(q_t, u_t, w_t), \forall T
\]
\[
\Rightarrow \sum_{t=0}^{T} \sigma_{\tau, R_2}(q_t, u_t, w_t) \geq \inf_{T \geq 0} \sum_{t=0}^{T} \sigma_{\tau, R_1}(q_t, u_t, w_t), \forall T
\]
\[
\Rightarrow \inf_{T \geq 0} \sum_{t=0}^{T} \sigma_{\tau, R_2}(q_t, u_t, w_t) \geq \inf_{T \geq 0} \sum_{t=0}^{T} \sigma_{\tau, R_1}(q_t, u_t, w_t).
\]
Since \((\tau, R_1)\) is feasible, there exists a controller \(\phi : Q \to \mathcal{U}\) such that (4) holds. We thus have
\[
\inf_{T} \sum_{t=0}^{T} \sigma_{\tau,R_1}(q_t, \phi(q_t), w_t) > -\infty
\]
\[
\Rightarrow \inf_{T} \sum_{t=0}^{T} \sigma_{\tau,R_2}(q_t, \phi(q_t), w_t) > -\infty
\]
and \((\tau, R_2)\) is also feasible.

Proposition 1 can be visualized in Fig 2. Specifically, for a fixed \(\tau > 0\), the range of values of \(R\) such that the pair \((\tau, R)\) is feasible is an interval of the form \([0, R^*(\tau)]\) where \(R^* : \mathbb{R}_+ \to \mathbb{R}_+\) is defined by
\[
R^*(\tau) = \max\{R \geq 0 | (\tau, R) \text{ is feasible}\}. \tag{7}
\]

\(R^*(\tau)\) is represented as a blue line in Figure 2. The problem under consideration can now be thought of as computing
\[
\tau^* = \arg \max_{\tau} R^*(\tau)
\]
with \(R^*(\tau^*)\) being the desired optimal solution.

![Fig. 2. Visualization of the feasible parameter space \(R - \tau\)](image)

**Proposition 2.** Given \(M\) as in (1), a performance objective (2) and a choice of \(R \geq 0\). Assume that the input sequences \(\{u_t\}\) and \(\{w_t\}\) satisfy inequality (3). Then: If \((\tau_1, R)\) is feasible for some \(\tau_1 > 0\), then \((\tau_2, R)\) is feasible for any \(0 < \tau_2 < \tau_1\).

**Proof.** Since \((\tau_1, R)\) is feasible, there exists a controller \(\phi : Q \to \mathcal{U}\) such that the closed loop system \((M, \phi)\) satisfies (2). Let \(\{q_t\}\) be the state trajectory of \((M, \phi)\) starting from initial state \(q_0 = q_o\) under an exogenous input sequence \(\{w_t\}\). We have
\[
\inf_{T} \sum_{t=0}^{T} \sigma_{\tau_1,R}(q_t, \phi(q_t), w_t) > -\infty,
\]
and hence
\[
\sum_{t=0}^{T^*} \sigma_{\tau,R}(q_t, \phi(q_t), w_t) > -\infty, \forall T^* \in \mathbb{Z}_+ \tag{8}
\]

Pick a choice of \(T^* \in \mathbb{Z}_+\). Under assumption (3), we also have
\[
\inf_{T \geq 0} \sum_{t=0}^{T} \sigma_{\tau,R}(q_t, \phi(q_t), w_t) > -\infty
\]
\[
\Rightarrow \sum_{t=0}^{T^*} \sigma_{\tau,R}(q_t, \phi(q_t), w_t) > -\infty.
\]

Adding (8) and (9) we get
\[
\sum_{t=0}^{T^*} \sigma_{\tau,R}(q_t, \phi(q_t), w_t) > -\infty.
\]

Since the choice of \(T^*\) was arbitrary, we have
\[
\inf_{T \geq 0} \sum_{t=0}^{T} \sigma_{\tau,R}(q_t, \phi(q_t), w_t) > -\infty.
\]
and hence \((\tau_2, R)\) is also feasible. \(\square\)

**Theorem 2.** Given \(M\) as in (1) and a performance objective (2). Assume that the input sequences \(\{u_t\}\) and \(\{w_t\}\) satisfy inequality (3). Then function \(R^* : \mathbb{R}_+ \to \mathbb{R}_+\) defined in (7) is monotonically non-increasing.

**Proof.** It suffices to show \(R^*(\tau_1) \geq R^*(\tau_2)\) for any \(\tau_1 < \tau_2\). Indeed, fixing \(\tau_2\) and applying Proposition 1, we can compute \(R^*(\tau_2)\) such that \((\tau_2, R^*(\tau_2))\) is feasible. Next, fixing \(R^*(\tau_2)\) and applying Proposition 2, we conclude that \((\tau_1, R^*(\tau_2))\) is also feasible, which implies that \(R^*(\tau_1) \geq R^*(\tau_2)\). \(\square\)

Theorem 2 provides an analytical solution for the problem under consideration. Indeed, under the stated assumptions on the inputs, the largest value of \(R\) is obtained in the limit as \(\tau \to 0\).

V. A NETWORK PERSPECTIVE

It was shown through a specific construction presented in [10] that one can view a DFM as a network, leading to a more intuitive interpretation of DFM analysis. In this Section, we revisit our main result and explain it from a graph theoretic viewpoint so as to provide our readers with a better understanding of the control design synthesis procedure considered in this paper.

A. Network Construction & a Relevant Result

A directed graph \(G = (N, E)\) consists of a vertex set \(N = \{1, \ldots, n\}\) and a set of directed edges \(E \subset N \times N = N^2\). \((i, j) \in E\) is a directed edge from vertex \(i\) to vertex \(j\). A network \(G = (N, E, c)\) is a directed graph with cost \(c_{ij}\) associated with each edge \((i, j)\). A path is a sequence of edges \((i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)\). A path is simple if \(i_1, \ldots, i_k\) are all distinct. A cycle is a simple path such that
In particular, consider a DFM $M$ as in (1) with $Q = \{ q_1, q_2, \ldots, q_n \}$, and a given cost function $\sigma : Q \times U \times W \to \mathbb{R}$. We can associate with $M$ a network $G_M$ constructed as follows:

$$N := \{ 1, \ldots, n \}$$

$$E := \{(i, j) \in N^2 | \exists (u, w) \in U \times W \text{ such that } q_j = f(q_i, u, w)\}$$

$$c_{ij} := \min_{(u, w) | f(q_i, u, w) = q_j} \sigma(q_i, u, w)$$

By this construction, analyzing whether $M$ meets the performance objective defined by $\sigma$ can be translated to a network flow problem by the following result.

**Theorem 3.** (Adapted from Lemma 6 in [10]) $M$ satisfies the following inequality

$$\inf_{T \geq 0} \sum_{t=0}^{T} \sigma(q_t, u_t, w_t) > -\infty \quad (10)$$

iff $G_M$ has no negative cost cycles.

**B. Analysis From a Network Perspective**

In this Section, we provide a different interpretation of DFM control synthesis and analyze the problem under consideration from a network perspective. Specifically, we first interpret the control design procedure in the network setting, and then we provide an alternate proof for Theorem 2. We begin with an observation.

**Proposition 3.** Let $G_M$ and $G_{(M, \varphi)}$ be the networks, constructed as described in the previous Section, associated with $M$ and the closed-loop system $(M, \varphi)$, respectively. $G_{(M, \varphi)}$ is a sub-network of $G_M$.

**Proof.** Let $N_M, E_M$ be the vertex set and edge set of $G_M$ and $N_{(M, \varphi)}, E_{(M, \varphi)}$ be the vertex set and edge set of $G_{(M, \varphi)}$. Note that closing the loop around $M$ with $\varphi : Q \to U$ can be interpreted as removing edges from the corresponding network. Therefore $N_M = N_{(M, \varphi)}$ and $E_{(M, \varphi)} \subseteq E_M$. Moreover, the edges that remain maintain their cost. Consequently, $G_{(M, \varphi)}$ is a sub-network of $G_M$.

As a result, the problem of interest can be equivalently reformulated as the problem of removing the edges associated with all but one "control input" for each node, such that the resulting network $G_{(M, \varphi)}$ contains no negative cost cycles. The edge costs here are understood to be determined by $\sigma = \sigma_{\tau, R}$ as defined in (5).

Now we are ready to provide an alternative proof for Theorem 2.

**Proof.** Suppose there exists a full state feedback law $\varphi : Q \to U$ such that $(M, \varphi)$ meets performance objective (2) for some value of $\tau$ and $R$. Let $G_{(M, \varphi)}$ be the corresponding network. Let $C_1, C_2, \ldots, C_p$ be the costs of the simple cycles in $G_{(M, \varphi)}$, and let $l_1, l_2, \ldots, l_k$ be the lengths of each of these cycles. We have

$$C_k = -l_k R - \sum_{j=1}^{l_k} h(q_j^k) + \tau \sum_{j=1}^{l_k} \left( \mu_\Delta(u_j) - \gamma \rho_\Delta(u_j) \right)$$

where $\{ q_j^k \}_{j=1}^{l_k}$ are the states corresponding to the nodes visited by cycle $k$ and $\{ u_j \}_{j=1}^{l_k}$, $\{ w_j \}_{j=1}^{l_k}$ are the inputs driving these cycles. By Theorem 3, $C_k \geq 0$ for all $k \in \{ 1, \ldots, p \}$ for the given values of $\tau$ and $R$.

Now, let $R^*_k(\tau)$ denote the maximal value of $R$ such that the cost $C_k$ remains non-negative for the given value of $\tau$. We have

$$R^*_k(\tau) = -\frac{1}{l_k} \sum_{j=1}^{l_k} h(q_j^k) + \tau \frac{l_k}{l_k} \sum_{j=1}^{l_k} \left( \mu_\Delta(u) - \gamma \rho_\Delta(u) \right).$$

In addition, it follows from assumption (3) that

$$\sum_{j=1}^{l_k} \mu_\Delta(u) - \gamma \rho_\Delta(u) \leq 0.$$

Therefore, $R^*_k(\tau)$ is monotonically non-increasing for any choice of $k \in \{ 1, \ldots, p \}$. Hence $R^*_k(\tau) = \min(R^*_1(\tau), R^*_2(\tau), \ldots, R^*_p(\tau))$ is also monotonically non-increasing for this choice of controller. Finally, optimizing over all feasible choices of controllers, it follows that $R^*(\tau)$ is monotonically non-increasing.

VI. ILLUSTRATIVE EXAMPLES

A. A Counterexample

Consider the DFM $M$ shown in Figure 3. $M$ has binary states $Q = \{ q_1, q_2 \}$ and binary input sets $U = \{ 0, 1 \}$ and $W = \{ 0, 1 \}$. Its state transitions are defined as follows:

$$f(q, u, w) \begin{cases} f(q_1, 0, 0) = q_1 \\ f(q_1, 0, 1) = q_1 \\ f(q_1, 1, 0) = q_2 \\ f(q_1, 1, 1) = q_2 \\ f(q_2, 0, 0) = q_2 \\ f(q_2, 0, 1) = q_1 \\ f(q_2, 1, 0) = q_2 \\ f(q_2, 1, 1) = q_1 \end{cases} \quad (11)$$

![Fig. 3. A counterexample: DFM M](image-url)
The performance output is given by \( h(q_1) = 100, h(q_2) = -1000 \). Functions \( \rho_\Delta, \mu_\Delta \) are defined by \( \rho_\Delta(u) = 1, \mu_\Delta(w) = w \). We are given \( \gamma = 0.4 \). Therefore, \( \sigma_{\tau,R} \) defined in (5) can be rewritten as

\[
\sigma_{\tau,R}(q, u, w) = -R - h(q) + \tau(w - 0.4)
\]

By direct observation, there are three simple cycles, one of which has negative cost \((c_{11})\) for all values of \( R \geq 0 \) and \( \tau > 0 \). Successful feedback control design thereby needs to remove this cycle, requiring \( \nu(q_1) = 1 \). On the other hand, setting \( \nu(q_2) \) to 0 or 1 yields identical results since input \( w \) entirely determines the transition from state \( q_2 \).

The network associated with the resulting closed loop system thus has three cycles, of which two are simple namely cycle “a” given by (1,2),(2,1) and cycle “b” given by (2,2). The respective cycle costs are given by:

\[
\begin{align*}
C_a &= -2R + 900 + 0.2\tau \\
C_b &= -R + 1000 - 0.4\tau
\end{align*}
\]

Figure 5 plots the largest value of \( R \) for each choice of \( \tau \) that results in 0 cost for each of these cycles (shown in red for \( a \) and green for \( b \)): Note that the resulting \( R^*(\tau) \) (shown in black) is not monotonically non-increasing.

### B. An Example Demonstrating Computational Advantage

We revisit the illustrative example presented in [12], in which a sequence of \( \rho/\mu \) approximations were constructed and used for certified-by-design controller synthesis to stabilize the double integrator. We demonstrate, in the context of this illustrative example, the computational advantages of the solution proposed in this paper relative to the brute force sampling based approach.

Consider the setup shown in Fig. 6: Black lines represent analog systems and signals, while blue lines represent discrete systems and sampled signals, with sampling period \( T_s \). The plant (a double integrator) can be placed in feedback with one of the two static gains, \( k_1 \) and \( k_2 \). The problem here is to design a switching controller, based only on the information from a binary sensor, so as to stabilize the closed loop system and maximize its rate of exponential convergence \( \tilde{R} \).

We follow the systematic construction procedure presented in [12] to construct a sequence of state-based \( \rho/\mu \) approximations \( \{M_i\} \) for the plant [13], and we attempt control design by formulating an auxiliary control problem for the DFM approximation. The corresponding function \( \sigma_{\tau,R}(q, u, w) \) is given by:

\[
\sigma_{\tau,R}(q, u, w) = -R - \hat{v}(q, u) + \tau(w - \gamma), \quad \tau > 0
\]

where \( \gamma \in (0, 1) \). In other words, we have \( \mu_\Delta(w) = w \) and \( \rho_\Delta(u) = \gamma \), for some fixed value of \( \gamma \) between 0 and 1.

We present the results of two approaches for comparison: Specifically, we implemented our new approach presented in this paper setting \( \tau = 10^{-5} \), and we compared the results relative to the existing sampling based approach which examines 100 samples of the \( \tau \) parameter space. We report the data for the two approaches applied to several elements of the DFM approximation sequence, of increasing fidelity. We explain the significance of the reported parameters in Table I.

As shown in Table II, the computation time required to design the controller is reduced dramatically: The reduction...
is roughly 100 fold, which is in line with the fact that we are solving one minimax problem rather than a few hundred ones in the sampling based approach. Additionally, the provable rate of converges improves slightly, as we are now effectively finding a better suboptimal approximation of the optimal value.

We repeated the numerical experiment for smaller values of $\tau$, namely $\tau = 10^{-6}, 10^{-7}, 10^{-8}$. We noticed that further decreasing the value of $\tau$ does not change the simulation results, which leads us to believe we have reached the optimal value (within numerical error).

VII. Conclusion

We revisited the problem of designing a full-state feedback controller for a DFM so as to maximize a performance parameter $R$ subject to meeting a performance objective involving a scaling parameter $\tau$. Based on some simple observations and subject to additional assumptions on the inputs of the system, we presented a new, analytical approach for choosing the scaling parameter $\tau$ so as to optimize $R$. We also presented an intuitive, graph theoretic interpretation of our result. Finally, we demonstrated via numerical examples the resulting reduction in computation time.

VIII. Acknowledgments

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