Decentralized Hierarchical Control of Multiple Time Scale Systems

Danielle C. Tarraf and H. Harry Asada
d’Arbeloff Lab for Information Systems and Technology
Mechanical Engineering Department
Massachusetts Institute of Technology
Cambridge, MA 02139

Abstract
This paper exposes an idea for dual level control architecture for use in large-scale systems consisting of networks of smaller subsystems whose dynamical behavior exhibits multiple time scales. The proposed control scheme is decentralized in nature at the low level: The communication among subsystems is done indirectly through the high level control, whose design is based on a singularly perturbed model of the overall system.

1. Introduction
With the fast technological advances seen in the past century, it is no wonder that many control systems of interest nowadays are complex large-scale systems that can be viewed as a network of subsystems. Unfortunately, the traditional approach of one centralized controller per control system does not scale well, due to various reasons: on the one hand the increasingly complex behavior seen when various systems interact together and hence the inevitable uncertainty in modeling these systems is problematic [1], and on the other hand, the sheer dimensions of the system make the resulting controller design very difficult. Various approaches have been researched within the past few decades, most notably decentralized control schemes seem to work well when the subsystems are weakly coupled [2], and hierarchical control schemes have shown an inherent ability to simplify some highly complex control problems.

The systems of interest to us are large-scale networks of subsystems that have the additional property of exhibiting multiple time scale behavior. The goal of this paper is to provide an initial exposition of a control architecture envisioned for these systems, involving a combination of a hierarchical system wide control structure and decentralized schemes at the local level, and exploiting the multiple time scale behavior.

2. Problem Formulation
Consider a class of large-scale systems consisting of a network of subsystems, each with its own local actuators, sensors and control objectives. Hence, for a network of $n$ such systems, the equations of the subsystems are given by:

$$\dot{x}_i = A_{ij}x_i + B_{ij}u_j + \sum_{j=1}^{n} A_{ij}^C x_j$$

$$y_i = C_{ij}x_i + \sum_{j=1}^{n} C_{ij}^C x_j$$

(2.1)

(2.2)

The class of systems of interest has the additional property that the dynamics of the subsystems include both fast and slow phenomena. Hence, by introducing appropriate changes in the time scale, the dynamic equations (2.1) can be singularly perturbed and approximated by:

$$\dot{x}_i = \tilde{A}_{ij}x_i + \tilde{B}_{ij}u_j + \sum_{j=1}^{n} \tilde{A}_{ij}^C x_j + \sum_{j=1}^{n} \tilde{A}_{ij}^C z_j$$

$$\varepsilon \dot{z}_i = \tilde{E}_{ij}x_i + \tilde{E}_{ij}z_i + B_{ij}u_j + \sum_{j=1}^{n} \tilde{E}_{ij}^C x_j + \sum_{j=1}^{n} \tilde{E}_{ij}^C z_j$$

(2.3)

(2.4)

Figure 1: Configuration of Large Scale Systems of Interest
Moreover, the networks of interest are those that include an outer loop driven by a global actuator, in addition to the local inner loops driven by local actuators as shown in Figure 1. Many real-life systems fall within this general description, for instance building energy systems.

The objective is to devise a control architecture such that local control objectives (mainly tracking a reference signal) are met in such a way that some global performance criterion is simultaneously optimized. The only additional requirement is that any control scheme envisioned should have the ability to retain system stability and minimize degradation in performance criteria in the event one or more of the subsystems are abruptly turned off or on.

3. Proposed Approach

The proposed approach is the use of a dual level control scheme, with the controllers at different levels being based on different continuous models of the systems of interest. The higher-level controller would have a global though less detailed view of the overall system, and would strive to achieve the global optimization goal. The lower level controllers on the other hand would strive to achieve their local objectives in a manner consistent with the directives of the higher-level controllers. The proposed controller architecture is described schematically in Figure 2.

![Proposed Control Architecture](image)

In particular, the high-level control scheme is to be based on a reduced order model of the system given by equations (2.3)-(2.4), corresponding to the limiting case when \( \varepsilon \to 0 \). In other words, the high level controller is expected to operate at the slower time scale. This choice arises out of our expectation that the slower events will have more impact on a global level, while the faster dynamics will have a more localized effect. The role of the high-level control scheme is thus envisioned to include:

i) Intermediate path planning for the multiple subsystems in a manner that is consistent with the nature of the interactions between them and with the goal of optimizing the global performance criterion. Appropriately planned prescribed paths for some combination of states of each of the subsystems would enable us to minimize destructive interactions among subsystems as they seek to achieve their local goals.

ii) Control of the global actuator, such as to optimize the global criterion and to preserve the global stability of the system.

iii) Supervisory control of the system, including keeping track of which systems are turned on and off as well as additional safety features.

The low level controllers, on the other hand, will be expected to work in a decentralized manner, with each local controller being solely aware of its own inputs and measurements. The low level controllers are thus unable to communicate among each other except indirectly through the information they receive from the high level controller. The objectives of the low level controllers are threefold: achieving their local control objectives following the path prescribed to them by the high level controller stabilizing the local feedback subsystem.

4. Plans for the Future

While the general architecture of the proposed scheme is clear at this point, the details of the design are not. Much work is needed in order to make this scheme amenable to practical implementation. In particular, we will be focusing on the following issues:

- The paths prescribed by the high level controller are inevitably approximate due to the use of simplified models.
- A quantitative evaluation of the error is needed, which allows us to establish an acceptable error margin for the low level system in following the path prescribed to it.
- It is not clear how to make a proper choice of intermediate variables among the low and high levels of control. The choice of variables and corresponding paths should ensure that the subsystems are not given contradictory commands.
- Additionally, the state prescribed to a given system should be easily attainable.

References
