Price of Anarchy and Price of Information
in \( N \)-Person Linear-Quadratic Differential Games

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Motivation

Game theory vs. centralized optimization

- Global vs. Local
- Optimum vs. Inefficiency
- Economics vs. Engineering

How bad it is when

- agents act selfishly?
- agents do not have the capability of centralized planning?
Prisoner’s Dilemma: An Example

- The centralized optimization yields \((2^\circ, 2^\circ)\).
- The Nash equilibrium is \((1^*, 1^*)\).
- Loss of efficiency is 1/2.

<table>
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<tr>
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<th>(NC)</th>
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<tbody>
<tr>
<td>(NC)</td>
<td>(2^\circ, 2^\circ)</td>
<td>(0, 3)</td>
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<tr>
<td>(C)</td>
<td>(3, 0)</td>
<td>(1^<em>, 1^</em>)</td>
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In Search for a Metric

Price of Anarchy (PoA)

• is a metric to quantify such loss of efficiency.
• has been analyzed for special types of games, e.g. routing games, resource allocation games.
• allows to understand that simple mechanisms may work reasonably well.

Other Metrics:

• Price of Simplicity
• Price of Uncertainty
• Price of Fellowship
From Static to Dynamic Games

Extend PoA to differential games (DG):

- Agents aim to minimize their *long-term costs* subject to the controls of the players.
- Linear quadratic DGs.

Information structure (IS) matters:

- closed-loop feedback IS vs. open-loop IS
- *Price of Information* (Pol)
DG: Open-Loop

- Set of players: \( \mathcal{N} = \{1, 2, \cdots, N\} \).
- Each player chooses \( u_i(t) \) from his set of feasible controls \( U_i \subset \mathbb{R}^{m_i} \).
- The system state \( x(\cdot) \) evolves according to the differential equation

\[
\dot{x}(t) = f(x(t), u_1(t), \cdots, u_N(t), t), \quad x(0) = x_0,
\]

- Each player \( i \) seeks to minimize

\[
J_i(u) = \int_0^T F_i(x(t), u_1(t), \cdots, u_N(t), t) dt + S_i(x(T))
\]

- \( F_i \): player \( i \)'s positive instantaneous cost function.
- \( S_i \): positive terminal value function.
Information Structure (IS)

Let $\gamma_i \in \Gamma_i$ be policies for the players. The IS $\eta$ can be

- **Open loop (OL):** $u_i(t) = \gamma_i(t; x_0)$
- **Feedback (FB):** $u_i(t) = \gamma_i(t; x(t))$

Assumptions:

- $\gamma_i(t; \cdot)$ is Lipschitz in $x$.
- $f$ is Lipschitz in $x$ and $\{u_1, \ldots, u_N\}$ and jointly piecewise continuous.
Nash Equilibrium

Player $i$ is faced with the optimal control problem

$$(OC(i)) \min_{\gamma_i \in \Gamma^\eta_i} J_i(\gamma_i, \gamma^\eta_{-i}) := \int_0^T F_i(x, \gamma_i(\eta), \gamma^\eta_{-i}(\eta), t) dt + S_i(x(T))$$

s.t. $\dot{x}(t) = f(x, \gamma_i(\eta), \gamma^\eta_{-i}(\eta), t), \quad x(0) = x_0$.

**Definition 1.** For a DG with IS $\eta$, the policy $N$-tuple $\{\gamma^\eta_i, \ i \in \mathcal{N}\} =: \gamma^\eta$ is an $\eta$–Nash equilibrium if, for each $i \in \mathcal{N}$, $\gamma^\eta_i$ solves the optimal control problem $(OC(i))$. Let $\Gamma^\eta$ be the set of all $\eta$–Nash equilibria, as a subset of $\Gamma^\eta$.

- The *achieved values* of the objective functions of the players under a particular $\eta$–Nash equilibrium $\gamma^\eta$ is $J^\eta_i, i \in \mathcal{N}$.
- The *total cost achieved* is $J^\eta_\mu = \sum_{i \in \mathcal{N}} \mu_i J^\eta_i$. 
Centralized Optimization

The team problem under a centralized control:

\[
\text{(COC)} \min_{\gamma \in \Gamma} \sum_{i=1}^{N} \mu_i \left\{ \int_{0}^{T} F_i(x(t), \gamma(\eta), t) dt + S_i(x(T)) \right\}.
\]

s.t. \( \dot{x}(t) = f(x, \gamma(\eta), t) \), \( x(0) = x_0 \),

- \( J^\circ_\mu \) is the optimal value, \textit{independent} of the IS.
- We \textit{necessarily} have \( 0 < J^\circ_\mu \leq J^{n^*}_\mu \).
Price of Anarchy

Definition 2. Consider an $N$-person DG as above and its associated optimal control problem (COC) with $J^\circ_\mu > 0$.

The price of anarchy for the DG is

$$\rho^\eta_{N,\mu,T} := \max_{\gamma^{\eta*} \in \Gamma^{\eta*}} \frac{J^{\eta*}_\mu}{J^\circ_\mu}$$

as the worst-case ratio of the total game cost to the optimum social cost.
Price of Information

Definition 3.

• Let $\eta_1$ and $\eta_2$ be two information structures.
• Consider two $N$-person DGs which differ only in terms of their ISs, with game 1 having IS $\eta_1$, and game 2 having $\eta_2$.
• Let the values of a particular $\mu$ convex combination of the objective functions be $J^\eta_1^*$ and $J^\eta_2^*$, respectively, achieved under the Nash equilibria $\gamma^{\eta_1^*}$ and $\gamma^{\eta_2^*}$.

The price of information between the two ISs (under cost minimization) is given by

$$\chi^{\eta_2}_{\eta_1}(\mu) = \frac{\max_{\gamma^{\eta_2^*} \in \Gamma^{\eta_2^*}} J^\eta_2^*}{\max_{\gamma^{\eta_1^*} \in \Gamma^{\eta_1^*}} J^\eta_1^*}.$$  \hspace{1cm} (2)
Scalar LQ Games

Each player $i$ minimizes the cost functional:

$$J_i = \int_0^\infty (q_i x_i^2(t) + r_i u_i^2(t)) \, dt, \quad i \in \mathcal{N},$$

(3)

where $u \in \mathbb{R}$, $r_i \in \mathbb{R}_{++}$, $q_i \in \mathbb{R}_{++}$ and $x_i(t) \in \mathbb{R}$ evolves according to linear system dynamics

$$\dot{x}(t) = ax(t) + \sum_{i=1}^{N} b_i u_i(t), \quad x(0) = x_0$$

(4)

- $x_0 \in \mathbb{R}$ is the initial value of the state
- $a \in \mathbb{R}$ and $b = [b_1, \ldots, b_N] \in \mathbb{R}^N$, $b_i \neq 0$, $\forall i \in \mathcal{N}$. 
Theorem 1. Let \( \{k_i, \ i \in \mathcal{N}\} \) solve the coupled set of algebraic Riccati equations

\[
2 \left( a - \sum_{i=1}^{N} s_i k_i \right) k_i + q_i + s_i k_i^2 = 0, \ i \in \mathcal{N} \tag{5}
\]
satisfying the stability condition \( a - \sum_{i=1}^{N} s_i k_i < 0 \), where \( s_i := b_i^2/r_i \).

Then, the \( N \)-tuple of policies

\[
\gamma_i^*(x) = -\frac{b_i}{r_i} k_i x, \ i \in \mathcal{N},
\]
constitutes a feedback NE, with the corresponding cost for Player \( i \) being \( J_i^* = k_i x_0^2 \). Furthermore, the positively weighed total cost is

\[
J^*_{\mu} = \bar{k} x_0^2, \quad \bar{k} = \sum_{i=1}^{N} \mu_i k_i.
\]

If the coupled algebraic Riccati equations do not admit a solution which is also stabilizing, then the DG does not have a feedback NE. \( \diamond \)
Eigenvalue Problem

Define new parameters for $i = 1, \cdots, N$:

- $\sigma_i = s_i q_i$
- $\sigma_{\text{max}} = \max_i \sigma_i$
- $p_i = s_i k_i$
- $\lambda = \sum_{i=1}^{N} p_i - a$

$$2 \left( a - \sum_{i=1}^{N} s_i k_i \right) k_i + q_i + s_i k_i^2 = 0,$$

$$\implies p_i^2 - 2\lambda p_i + \sigma_i = 0. \quad (6)$$
Eigenvalue Problem

Let $\Omega \subset \mathcal{N}$ be an index set. For every $\Omega \neq \emptyset$, we have

\[ \prod_{j \in \Omega} p_j^\lambda = \prod_{j \in \Omega} \left( \sum_{i=1}^{N} p_i - a \right) = \prod_{j \in \Omega} p_j \sum_{i \in \Omega} p_i + \prod_{j \in \Omega} p_j \sum_{i \notin \Omega} p_i - a \prod_{j \in \Omega} p_j \]

\[= \sum_{i \in \Omega} \prod_{j \in \Omega} p_j p_i + \sum_{i \notin \Omega} \prod_{j \in \Omega} p_j p_i - a \prod_{j \in \Omega} p_j \]

\[= \sum_{i \in \Omega} p_i^2 \prod_{j \in \Omega_{\neg i}} p_j + \sum_{i \notin \Omega} \prod_{j \in \Omega} p_j p_i - a \prod_{j \in \Omega} p_j \]

\[= \sum_{i \in \Omega} (2\lambda p_i - \sigma_i)^2 \prod_{j \in \Omega_{\neg i}} p_j + \sum_{i \notin \Omega} \prod_{j \in \Omega} p_j p_i - a \prod_{j \in \Omega} p_j \]

\[= 2\lambda \sum_{i \in \Omega} \prod_{j \in \Omega} p_j - \sum_{i \in \Omega} \sigma_i \prod_{j \in \Omega_{\neg i}} p_j + \sum_{i \notin \Omega} \prod_{j \in \Omega} p_j p_i - a \prod_{j \in \Omega} p_j \]

\[= 2n_{\Omega} \prod_{j \in \Omega} p_j^\lambda - \sum_{i \in \Omega} \sigma_i \prod_{j \in \Omega_{\neg i}} p_j + \sum_{i \notin \Omega} \prod_{j \in \Omega} p_j p_i - a \prod_{j \in \Omega} p_j. \]


\textbf{Eigenvalue Problem}

When $\Omega \neq \emptyset$,

\[
\prod_{j \in \Omega} p_j \lambda = \frac{1}{2n_{\Omega} - 1} \left\{ \sum_{i \in \Omega} \sigma_i \prod_{j \in \Omega \setminus i} p_j - \sum_{i \notin \Omega} \prod_{j \in \Omega} p_j p_i + a \prod_{j \in \Omega} p_j \right\}.
\]

When $\Omega = \emptyset$,

\[
\prod_{j \in \Omega} p_j \lambda := \lambda = \sum_{j=1}^{N} p_j - a.
\tag{8}
\]

Let $p = [1, p_1, p_2, \cdots, p_N, p_1 p_2, \cdots, p_1 p_N, p_2 p_3, \cdots, p_{N-1} p_N, \cdots, \prod_{i=1}^{N} p_i]^T$.

\[
\tilde{M} p = \lambda p.
\tag{9}
Feedback NE Computation

- \( D = \text{diag}\{1, s_1, s_2, \ldots, s_N, s_1s_2, \ldots, s_1s_N,s_2s_3, \ldots, s_{N-1}s_N, \ldots, \prod_{i=1}^{N} s_i\} \).
- \( p = Dk \)

Then

\[ \text{M}k = \lambda k, \text{ where } \text{M} := D^{-1}\tilde{\text{M}}D. \quad (10) \]

**Theorem 2.** Suppose \( \text{M} \) is a nondefective matrix with distinct eigenvalues. Let \((\lambda, k)\) be an eigenvalue-eigenvector pair such that \( \lambda \in \mathbb{R}_+ \) and \( \lambda > \sigma_{\text{max}} \). Then, a feedback NE

\[ \gamma^*_i(x) = -\frac{b_i}{r_i} k_i x, \quad i \in \mathcal{N}, \]

is yielded by \( k^* = 1^T k \) provided that the resulting solution is stabilizing, where \( 1 = [0, 1, \cdots, 1, 0, \cdots, 0]^T \) is a vector whose 2nd to \( N+1 \)-st entries are 1's.
Theorem 3. The optimal control problem (FOC) admits a unique feedback solution which is further stabilizing. The optimal policies are

\[
\gamma_i^\circ(x) = -\frac{b_i}{\mu_i r_i} \hat{k}_\mu x, \quad \hat{k}_\mu := \frac{a + \sqrt{a^2 + \bar{q}b}}{\bar{b}},
\]  

(11)

with \( \bar{b} := \sum_{i=1}^{N} (b_i^2/\mu_i r_i) \), and minimum cost is \( J_\mu^\circ = \hat{k}_\mu x_0^2 \).

The optimal control can also be expressed in open loop:

\[
u_i^\circ = -\frac{b_i}{\mu_i r_i} \hat{k}_\mu \Phi(t, 0)x_0,
\]

where \( \Phi(t, 0) \) is the unique solution to

\[
\dot{\Phi}(t, 0) = \left( a - \sum_{i=1}^{N} \frac{b_i^2}{\mu_i r_i} \hat{k}_\mu \right) \Phi(t, 0), \quad \Phi(0, 0) = 1.
\]
**Price of Anarchy**

**Theorem 4.** The PoA of the LQ feedback DG described by (3) and (4) is characterized by the following:

(i) Given a weight vector $\mu$, the PoA $\rho_\mu$ is equal to

$$\rho_{FB}^{\mu} = \max_{k \in \mathcal{K}} \left[ \mu^T k \right] / \hat{k},$$

where $\mu = [0, \mu^T, 0, \cdots, 0]^T$ and $\mathcal{K}$ is the set of all eigenvectors of the matrix $M$.

(ii) Suppose $\mu_i = \bar{\mu}_i := s_i / \sum_{j=1}^{N} s_j, i \in \mathcal{N}$. Then,

$$\rho_{\bar{\mu}}^{FB} \leq \left[ \varrho(M) + a \right] / \sum_{i=1}^{N} s_i \hat{k},$$

where $\varrho(M)$ is the spectral radius of $M$.

(iii) Let $\mu_{\text{max}}^s = \max_{i \in \mathcal{N}} \mu_i / s_i$. Given a weight vector $\mu$ that satisfies $\sum_{i=1}^{N} \mu_i = 1$, the PoA is bounded by

$$\rho_{\mu}^{FB} \leq \mu_{\text{max}}^s \left( \varrho(M) + a \right) / \hat{k}.$$
Corollary 5.  The following two results follow Theorem 4:

(i) Given a $\mu$ and $a \neq 0$, the price of anarchy is bounded from above by

$$\rho_\mu \leq \left(1 + \frac{1}{2a}(N + \sigma_{\max} - 1)\right)s^\bullet,$$

where $\sigma_{\max} = \max_{i \in N} \sigma_i$ and $s^\bullet := \sum_{i=1}^{N} \min_{i \in N} \frac{s_i}{s_i}$. The upper-bound is independent of the choice $\mu$.

(ii) If $a = 0$, The price of anarchy is bounded from above by

$$\rho_\mu \leq \frac{\mu_{\max}^s}{\sqrt{q}} \sqrt{\mu_{\min}^s} \sqrt{N(N + \sigma_{\max} - 1)},$$

where $\mu_{\min}^s = \min_{i \in N} \mu_i/s_i$. 

Theorem 6. Suppose the number of players in the LQ DG is sufficiently large so that

\[(C-i) \ p_{-i} > a, \forall i \in \mathcal{N}, \ (C-\text{ii}) \ a \ll N, \ (C-\text{iii}) \ \sigma_{\text{max}} \ll \bar{\sigma},\]

where \(\bar{\sigma} = \sum_{i=1}^{N} \sigma_i\). Then, the following quantities can be approximated as given:

\(i) \ p_i \sim \frac{\sigma_i}{\sqrt{2\bar{\sigma}}}, \ (ii) \ u_i \sim -\frac{\sigma_i}{b_i \sqrt{2\bar{\sigma}}}x,\)

\(iii) \ J^* \sim \frac{\bar{q}}{\sqrt{2\bar{\sigma}}}(x_0)^2, \ (iv) \ J^* \sim \frac{\bar{q}}{\sqrt{2\bar{\sigma}}}(x_0)^2,\)

\(v) \ \rho_{FB}^\mu \sim \frac{\bar{q}}{k \sqrt{2\bar{\sigma}}}, \text{ and for } a = 0, \ \rho_{FB}^\mu \sim \sqrt{\frac{\bar{q}b}{2\bar{\sigma}}}.\)
Open-Loop LQ Games

Theorem 7. Consider the $N$-person LQ DG in (3) and (4), and assume that there exists a unique solution $\xi^*$ to the set of equations

$$0 = 2a\xi_i + q_i - \xi_i \left( \sum_{j=1}^{N} s_j \xi_j \right),$$

such that $a - \sum_{j=1}^{N} s_j \xi_j^* < 0$. Then the game admits an open-loop Nash equilibrium for every initial state, given by

$$u_i^*(t) = -\frac{b_i}{r_i} \xi_i^* \exp \left[ \left( a - \sum_{j=1}^{N} s_j \xi_j^* \right) t \right] x_0.$$

The optimal cost to player $i$ using $u_i^*$ is $J_i^* = k_i^* x_0$, where $k_i^*$ is the unique solution to

$$2 \left( a - \sum_{j=1}^{N} s_j \xi_j^* \right) k_i + q_i + s_i(\xi_i^*)^2 = 0. \tag{16}$$
Price of Information

Pol between open-loop and feedback ISs is defined by

\[ \chi_{OL}^{OFB} = \max_{k^*} \frac{J_{OL}^{*}}{J_{FB}^{*}}. \] (17)

When the feedback NE is unique, we have the expression

\[ \chi_{OL}^{OFB} = \frac{\rho_{OL}^{*}}{\rho_{FB}^{*}}. \]

- A bound on PoA leads to a bound on Pol.
Theorem 8. Suppose $a = 0$, and the number of players is large so that $N$ satisfies (C-i), (C-ii), and (C-iii). Then, the Pol is bounded from above and below by two constants:

$$\frac{\sqrt{2}}{2} \leq \chi_{OL}^{FB} \leq \sqrt{2}.$$  \hspace{1cm} (18)

Corollary 9. Suppose the DG satisfies the conditions in Theorem 8. In addition, let the players be symmetric so that $\sigma_i = \sigma, p_i = p, \forall i \in \mathcal{N}$. When $N \geq 3$, the open-loop IS yields better total optimal cost; otherwise the FB information does better. In addition, as $N \to \infty$,

$$\lim_{N \to \infty} \chi_{FB}^{OL} = \frac{\sqrt{2}}{2}$$

at the rate of $O\left(\frac{1}{N}\right)$. 


Multi-user Rate-Based Flow Control

- The traffic of source $i$ has an available bandwidth of $w_i$.
- Perfect measurement of the queue length $q_l(t)$.
- $s_r(t)$ is total effective service rate available at the link.
- $w_is_r(t)$ is the available bandwidth to source $i$.
- $d_i(t)$ is the rate of source $i$ at time $t$ and $u_i(t) := d_i(t) - w_is_r(t)$.
- The dynamics for the queue length

\[
\dot{q}_l(t) = \sum_{i=1}^{N} u_i(t). \quad (19)
\]
• Ensure the bottleneck queue size stays around some desired level $\bar{q}_l$.

• Let $x(t) := q_l(t) - \bar{q}_l$. Consider a shifted version of (19) as follows.

\[
\dot{x}(t) = \sum_{i=1}^{N} u_i, x(0) = x_0
\]  

(20)

Each source minimizes its own cost:

\[
J_i(u) = \int_0^{\infty} \left( |x(t)|^2 + \frac{1}{c_i} |u_i(t)|^2 \right),
\]  

(21)

Related team problem

\[
J(u) = \int_0^{\infty} \left( M |x(t)|^2 + \sum_{i=1}^{N} \frac{1}{c_i} |u_i(t)|^2 \right).
\]  

(22)
Set the parameters in the model (3) and (4):

- $c_i = 1$,
- $a = 0, x_0 = 1$,
- $\sigma_i = s_i = q_i = r_i = b_i = 1$.

\[ M_2 : \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1/3 & 1/3 & 0 \end{bmatrix}, M_3 : \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & -1/3 \\ 0 & 1/3 & 0 & 1/3 & 0 & 0 & 0 & -1/3 \\ 0 & 0 & 1/3 & 1/3 & 0 & 0 & 0 & -1/3 \\ 0 & 0 & 0 & 0 & 1/5 & 1/5 & 1/5 & 0 \end{bmatrix} \]
Normalized Dynamics

We introduce a normalization factor is to adjust the queue length proportionally when the number of users increases

\[
\dot{x}(t) = \frac{1}{f(N)} \sum_{i=1}^{N} u_i, \quad x(0) = x_0
\]  

(23)

**Theorem 10.** The prices of anarchy \( \rho^\mu_{OL} \), \( \rho^\mu_{FB} \) and the price of information \( \chi^\mu_{FB} \) are independent of normalization factor \( f(N) \). The results with the normalization factor are summarized in the following table.
### Result Summary

<table>
<thead>
<tr>
<th>$J^*$ (FB)</th>
<th>$J^\circ$ (TP)</th>
<th>$J^*$ (OL)</th>
<th>$\rho^F_B$</th>
<th>$\rho^O_L$</th>
<th>$\chi^O_{FB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{f(N)}{\sqrt{2N-1}}$</td>
<td>$\frac{f(N)}{N}$</td>
<td>$\frac{f(N)}{\sqrt{N}} \left( \frac{1}{2} + \frac{1}{2N} \right)$</td>
<td>$\frac{N}{\sqrt{2N-1}}$</td>
<td>$\sqrt{N} \left( \frac{N+1}{2N} \right)$</td>
<td>$\sqrt{2 - \frac{1}{N} \left( \frac{1}{2} + \frac{1}{N} \right)}$</td>
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### Result Summary Under Large Population Approximation

<table>
<thead>
<tr>
<th>$f(N)$</th>
<th>$J^*$ (FB)</th>
<th>$J^\circ$ (TP)</th>
<th>$J^*$ (OL)</th>
<th>$\rho^F_B$</th>
<th>$\rho^O_L$</th>
<th>$\chi^O_{FB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$\frac{1}{\sqrt{2N}}$</td>
<td>$\frac{1}{N}$</td>
<td>$\frac{1}{\sqrt{N}} \left( \frac{1}{2} + \frac{1}{2N} \right)$</td>
<td>$\sqrt{\frac{N}{2}}$</td>
<td>$\sqrt{N} \left( \frac{1}{2} + \frac{1}{2N} \right)$</td>
<td>$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2N}$</td>
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<td>$\frac{1}{N}$</td>
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</tr>
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Conclusion and Future Work

• PoA is an important metric for the purpose of design:
  – How bad is selfish behavior?
  – How to control the performance? (via access control, pricing, etc.)
• PoI is an important metric to quantify the value of information in terms of performance.
  – “Less is More”?
  – How to design a good system with least information structure? (v.s. issue of robustness)
• PoA of a Stackelberg LQ games, e.g. security game, jammer is the leader and the others are followers in the passive defense.
